

Electrical Circuits and Network (313332)

MSBTE K-Scheme · Model Question Paper — Set B · Model Answers

Q.1

a) Active power and reactive power.

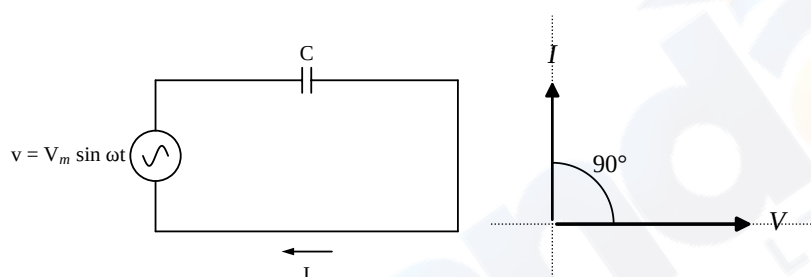
2 M

- **Active (real) power P:** the power actually consumed in the resistive part of the circuit, $P = VI \cos \phi = I^2 R$. Unit: watt (W).
- **Reactive power Q:** the power that oscillates between the source and the reactive elements (L, C) without being consumed, $Q = VI \sin \phi$. Unit: volt-ampere reactive (VAR).

(Each definition with unit 1 M)

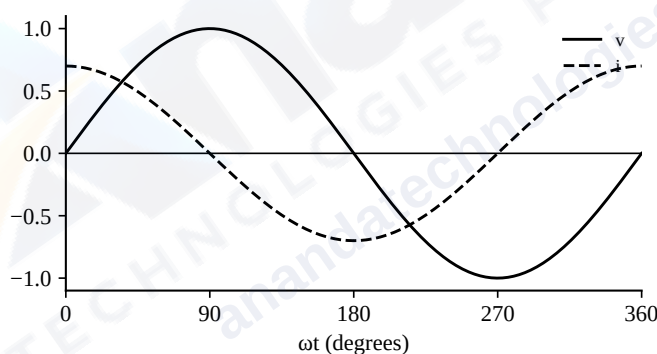
b) Purely capacitive circuit — waveforms and phasor diagram.

2 M



(a) Circuit

(c) Phasor diagram



(b) Waveforms: i leads v by 90°

- Applied voltage $v = V_m \sin \omega t$; charge $q = Cv$, so $i = C dv/dt = \omega C V_m \cos \omega t$.
- Current $i = I_m \sin(\omega t + 90^\circ)$, where $I_m = V_m/X_C$.
- Capacitive reactance $X_C = 1/(\omega C) = 1/(2\pi f C)$ ohm; current **leads** voltage by 90° .
- Power factor = $\cos 90^\circ = 0$ (leading); average power $P = 0$ — energy is stored in the electric field and returned.

(Waveforms 1 M, phasor diagram 1 M)

c) Q-factor of series resonant circuit.

2 M

- **Q-factor (quality factor):** In a series resonant circuit it is the voltage magnification at resonance: $Q = V_L/V = V_C/V = \omega_r L/R = (1/R)\sqrt{L/C}$. It is a measure of the selectivity

(sharpness) of the resonance curve. $Q = f_r/BW$.

(Definition 1 M, expression 1 M)

d) Advantages of 3-phase over 1-phase.

2 M

1. For the same power and voltage, a 3-phase system needs less conductor material (about 75 % of 1-phase).
2. 3-phase power is constant (non-pulsating); 1-phase power pulsates at twice the supply frequency.
3. 3-phase induction motors are self-starting; 1-phase motors are not self-starting.
4. For the same frame size, 3-phase machines have higher output and better efficiency and power factor.

(Any four, $\frac{1}{2}$ M each)

e) Internal resistance of ideal sources.

2 M

- Ideal voltage source: internal resistance = **zero** ($r = 0$).
- Ideal current source: internal resistance = **infinite** ($r = \infty$).

(1 M each)

f) Superposition theorem.

2 M

In a linear, bilateral network containing more than one source, the current (or voltage) in any branch is the **algebraic sum** of the currents (or voltages) produced in that branch by each source acting **alone**, all other sources being replaced by their internal resistances (voltage sources short-circuited, current sources open-circuited).

(Statement 2 M)

g) Reciprocity theorem.

2 M

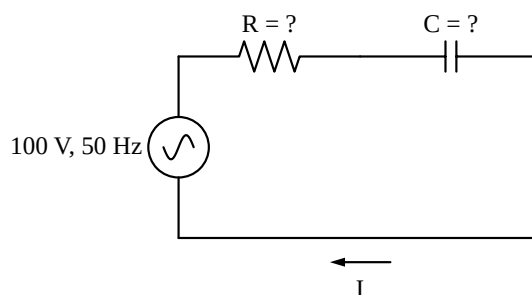
In any linear, bilateral, passive network containing a single source, if an emf E in branch 1 produces a current I in branch 2, then the same emf E placed in branch 2 will produce the **same current I** in branch 1. The ratio E/I (transfer resistance) remains the same when the positions of source and ammeter are interchanged.

(Statement 2 M)

Q.2

a) R-C series: 100 V, 50 Hz, $I = 2.5$ A, $P = 150$ W.

4 M



Circuit diagram

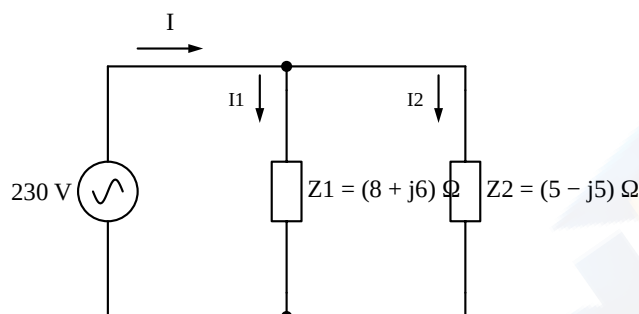
1. Impedance $Z = V/I = 100/2.5 = 40 \Omega$
2. Power $P = I^2 R \Rightarrow R = P/I^2 = 150/2.5^2 = 24 \Omega$
3. Reactance $X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(40^2 - 24^2)} = 32 \Omega$
4. $C = 1/(2\pi f X_C) = 1/(2\pi \times 50 \times 32) = 99.472 \mu F$
5. Power factor $\cos \phi = R/Z = 0.6$ (leading)

(Z 1 M, R 1 M, X 1 M, C 1 M)

R = 24 Ω , C = 99.472 μF , p.f. = 0.6 leading

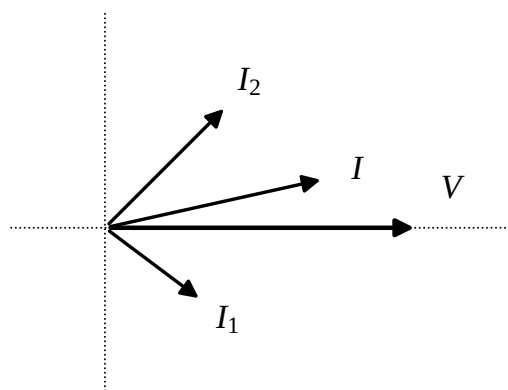
b) Admittance method: $(8 + j6) \Omega \parallel (5 - j5) \Omega$ across 230 V.

4 M



Circuit diagram

1. $Z_1 = (8 + j6) \Omega = 10 \angle 36.87^\circ \Omega$
2. $Z_2 = (5 - j5) \Omega = 7.071 \angle -45^\circ \Omega$
3. $Y_1 = 1/Z_1 = 1/10 \angle 36.87^\circ = 0.1 \angle -36.87^\circ S = (0.08 - j0.06) S$, i.e. $G_1 = 0.08 S$, $B_1 = 0.06 S$
4. $Y_2 = 1/Z_2 = 1/7.071 \angle -45^\circ = 0.141 \angle 45^\circ S = (0.1 + j0.1) S$, i.e. $G_2 = 0.1 S$, $B_2 = 0.1 S$
5. Total admittance $Y = Y_1 + Y_2 = (0.18 + j0.04) S = 0.184 \angle 12.53^\circ S$
6. Branch current $I_1 = V/Z_1 = 230 \angle 0^\circ / 10 \angle 36.87^\circ = 23 \angle -36.87^\circ A = (18.4 - j13.8) A$
7. Branch current $I_2 = V/Z_2 = 230 \angle 0^\circ / 7.071 \angle -45^\circ = 32.527 \angle 45^\circ A = (23 + j23) A$
8. Total current $I = V \times Y = 230 \times 0.184 \angle 12.53^\circ = 42.41 \angle 12.53^\circ A = (41.4 + j9.2) A$
1. p.f. of branch 1 = $\cos \phi_1 = R_1/|Z_1| = 8/10 = 0.8$ lagging
2. p.f. of branch 2 = $\cos \phi_2 = R_2/|Z_2| = 5/7.0711 = 0.7071$ leading
3. Overall p.f. = $\cos(\text{angle between } V \text{ and } I) = \cos 12.53^\circ = 0.9762$ leading



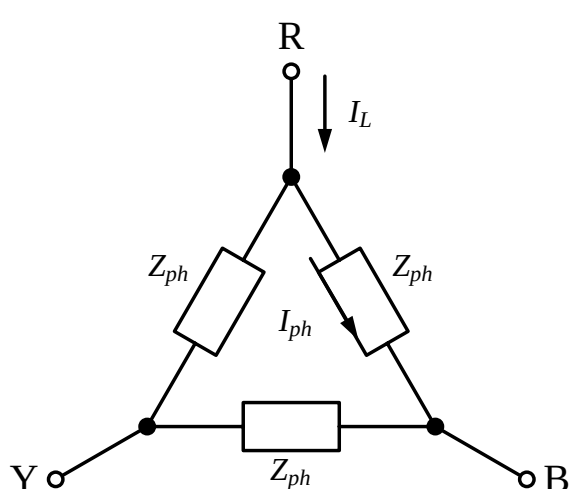
Phasor diagram (V as reference)

(Branch currents 2 M, total current/p.f. 2 M, power/phasor diagram 2 M)

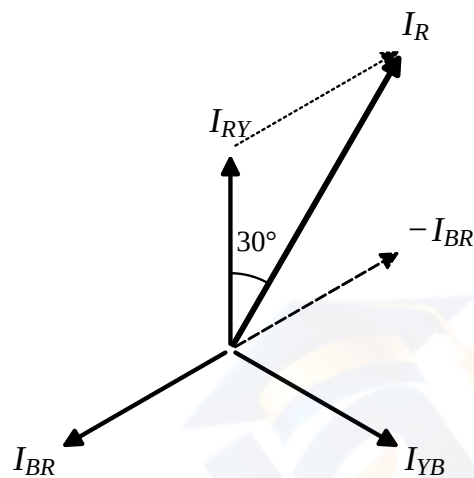
$$I_1 = 23 \angle -36.87^\circ \text{ A}, I_2 = 32.527 \angle 45^\circ \text{ A}, I = 42.41 \angle 12.53^\circ \text{ A}, \text{p.f.} = 0.9762 \text{ leading}$$

c) Derive $I_L = \sqrt{3} I_{ph}$ for delta connection.

4 M



Delta connected load



Phasor diagram ($I_R = I_{RY} - I_{BR}$)

- In delta connection each phase is connected directly between two lines, so $V_L = V_{ph}$.
- By KCL at junction R: line current $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$.
- I_{RY} and $-I_{BR}$ are equal (I_{ph}) and 60° apart, so $I_R = 2 I_{ph} \cos 30^\circ = 2 I_{ph}(\sqrt{3}/2)$
- $I_L = \sqrt{3} I_{ph}$; similarly for I_Y and I_B . Line currents lag the respective phase currents by 30° .

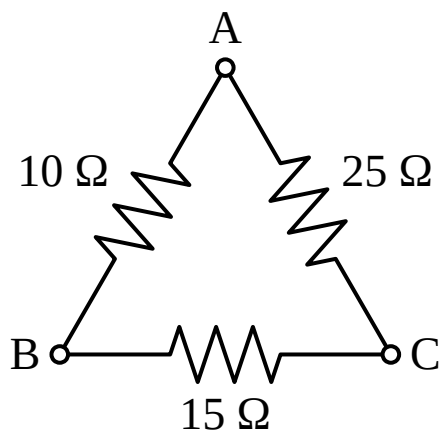
(Diagram 1 M, phasor diagram 1 M, derivation 2 M)

d) Delta to star conversion ($R_{AB} = 10 \Omega$, $R_{BC} = 15 \Omega$, $R_{CA} = 25 \Omega$).

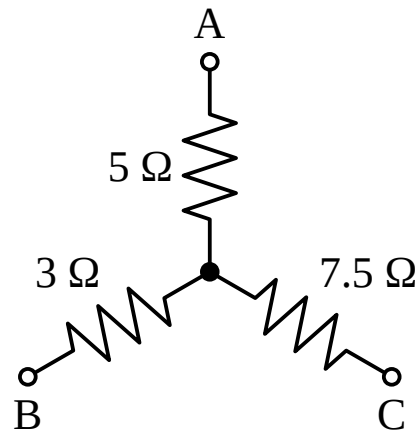
4 M

Sum of delta resistances = $R_{AB} + R_{BC} + R_{CA} = 10 + 15 + 25 = 50 \Omega$

1. $R_A = R_{AB}R_{CA}/\Sigma R = (10 \times 25)/50 = 5 \Omega$
2. $R_B = R_{AB}R_{BC}/\Sigma R = (10 \times 15)/50 = 3 \Omega$
3. $R_C = R_{BC}R_{CA}/\Sigma R = (15 \times 25)/50 = 7.5 \Omega$



Given delta



Equivalent star

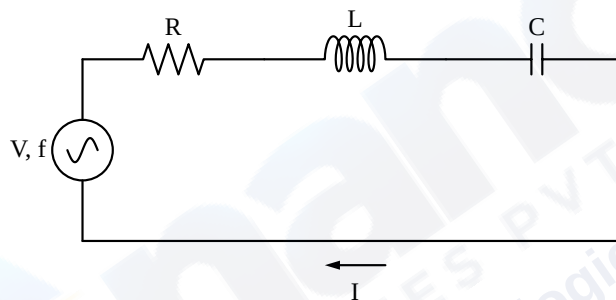
Check: resistance between A and B — delta: $10 \parallel (15 + 25) = 8 \Omega$; star: $5 + 3 = 8 \Omega$ ✓

(Formulae 1 M, three values 3 M)

Q.3

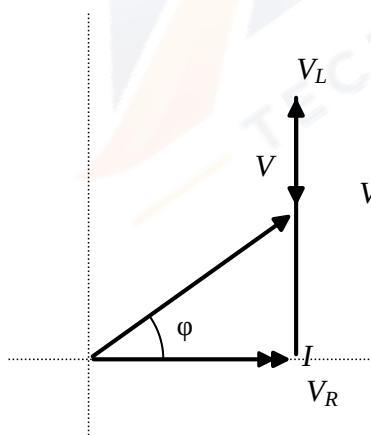
a) Impedance and voltage triangles of series R-L-C.

4 M

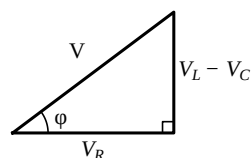


Series R-L-C circuit

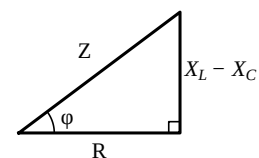
Case i) $X_L > X_C$ — circuit is inductive, current lags voltage, p.f. lagging:



Phasor diagram

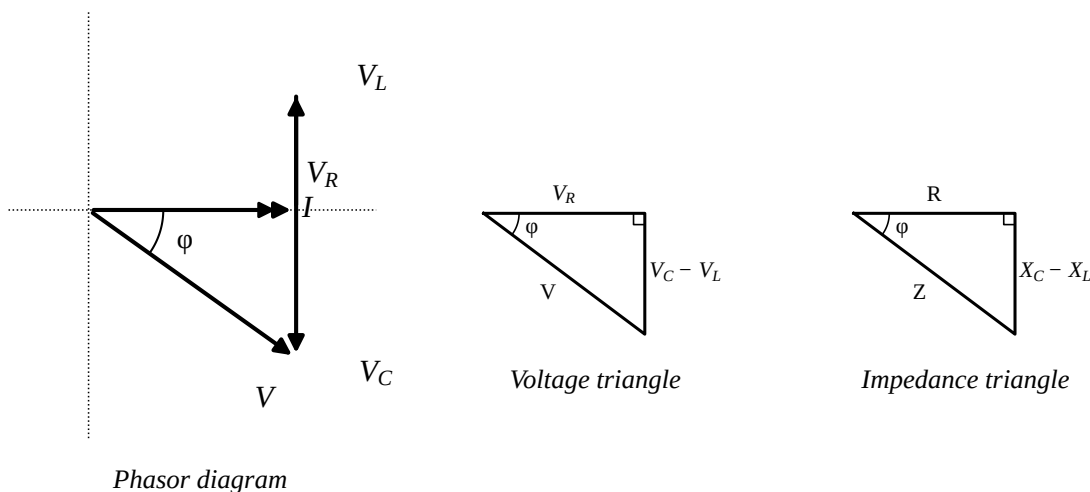


Voltage triangle



Impedance triangle

Case ii) $X_C > X_L$ — circuit is capacitive, current leads voltage, p.f. leading:



Case iii) $X_L = X_C$ — resonance: $Z = R$, $V = V_R$, current in phase with voltage, p.f. = 1.

(Each case 2 M)

b) Features of parallel resonance and resonance curve.

4 M

- At resonance the circuit behaves as a pure resistance $Z_D = L/CR$; power factor is unity.
- Impedance is **maximum** and the line current is **minimum** ($I = V/Z_D$); so it is called a **rejector circuit**.
- A large current circulates between L and C (current magnification = Q).
- Resonant frequency $f_r = (1/2\pi)\sqrt{(1/LC - R^2/L^2)}$.
- Used in oscillator tank circuits, IF amplifiers, filters and wave-traps.

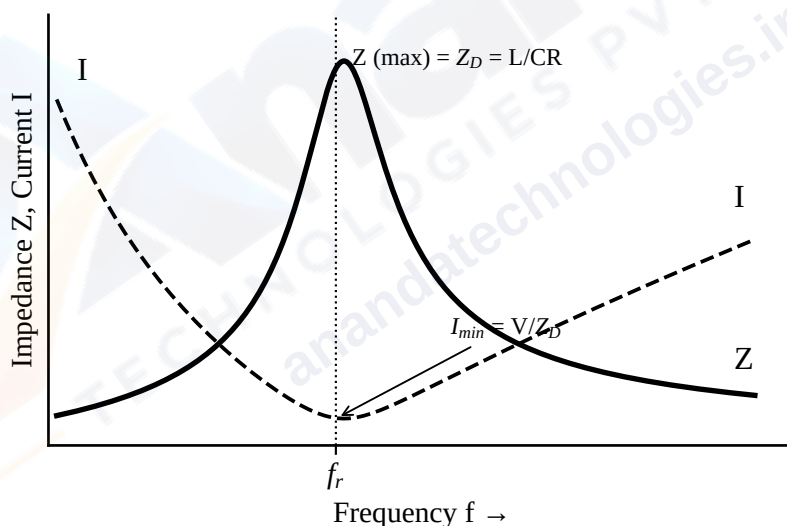


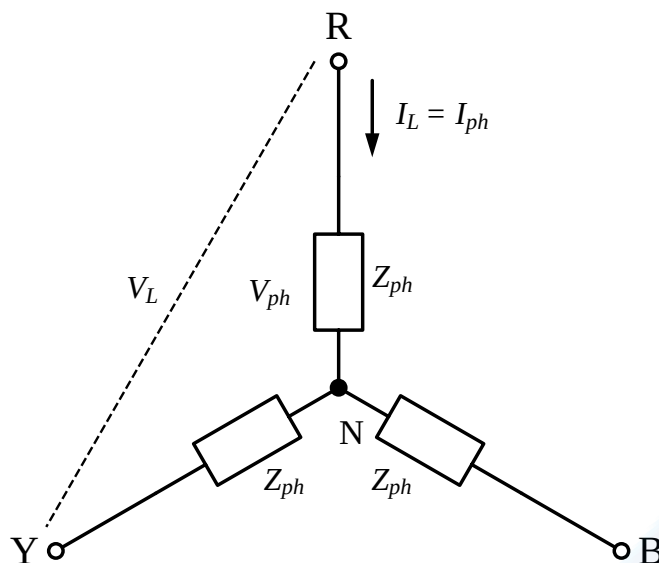
Fig.: Resonance curve of parallel circuit — Z and I versus frequency

- **Z curve:** impedance rises to maximum Z_D at f_r , falls on both sides.
- **I curve:** line current falls to minimum $I_{min} = V/Z_D$ at f_r .
- **f_r :** resonant frequency; below f_r the circuit is inductive, above f_r capacitive.

(Features 2 M, curve with terms 2 M)

c) Star load: 15 kW, 415 V, $I_L = 25$ A.

4 M



Star connected load

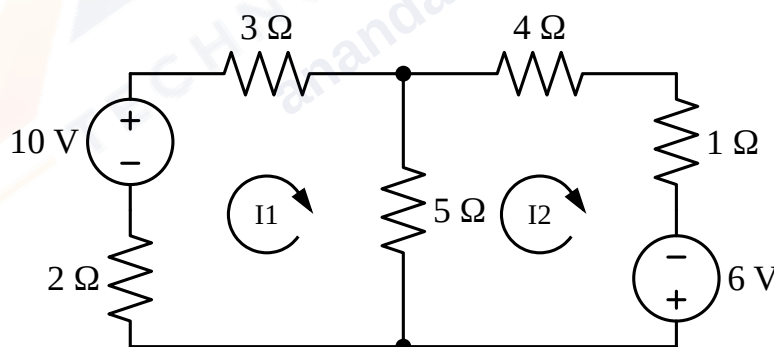
1. $P = \sqrt{3} V_L I_L \cos \phi \Rightarrow \cos \phi = P/(\sqrt{3} V_L I_L) = 15000/(\sqrt{3} \times 415 \times 25) = \mathbf{0.8347 \text{ lagging}}$
2. $V_{ph} = V_L/\sqrt{3} = 239.6$ V; $I_{ph} = I_L = 25$ A (star)
3. $Z_{ph} = V_{ph}/I_{ph} = 239.6/25 = \mathbf{9.584 \Omega}$
4. $R_{ph} = Z_{ph} \cos \phi = 9.584 \times 0.8347 = \mathbf{8 \Omega}$
5. $X_{ph} = Z_{ph} \sin \phi = 9.584 \times 0.5507 = \mathbf{5.2776 \Omega}$ (inductive)
6. Inductance per phase $L = X_L/(2\pi f) = 5.2776/(2\pi \times 50) = \mathbf{0.0168 \text{ H}}$

(p.f. 1 M, Z_{ph} 1 M, R_{ph} 1 M, X_{ph} / L or C 1 M)

p.f. = 0.8347 lag, $R_{ph} = 8 \Omega$, $X_{ph} = 5.278 \Omega$, $L = 16.8$ mH

d) Mesh analysis — current in 5 Ω (Fig. No. 2).

4 M



Mesh currents assumed clockwise

Assume clockwise mesh currents I_1 , I_2 as shown. Applying KVL to each mesh (voltage drops = emf in the mesh):

Mesh 1: $2 I_1 + 3 I_1 + 5(I_1 - I_2) = 10$

Mesh 2: $5(I_2 - I_1) + 4 I_2 + 1 I_2 = 6$

Simplifying:

$$10 I_1 - 5 I_2 = 10 \quad \dots(1)$$

$$-5 I_1 + 10 I_2 = 6 \quad \dots(2)$$

Solving by Cramer's rule: $\Delta = \begin{vmatrix} 10 & -5 \\ -5 & 10 \end{vmatrix} = 75$

$$\Delta_1 = \begin{vmatrix} 10 & -5 \\ 6 & 10 \end{vmatrix} = 130 \Rightarrow I_1 = \Delta_1 / \Delta = 130 / 75 = \mathbf{1.7333 \text{ A}}$$

$$\Delta_2 = \begin{vmatrix} 10 & 10 \\ -5 & 6 \end{vmatrix} = 110 \Rightarrow I_2 = \Delta_2 / \Delta = 110 / 75 = \mathbf{1.4667 \text{ A}}$$

Current in 5 Ω resistor (common to mesh 1 and mesh 2) = $I_1 - I_2 = 1.7333 - (1.4667) = \mathbf{0.2667 \text{ A}}$

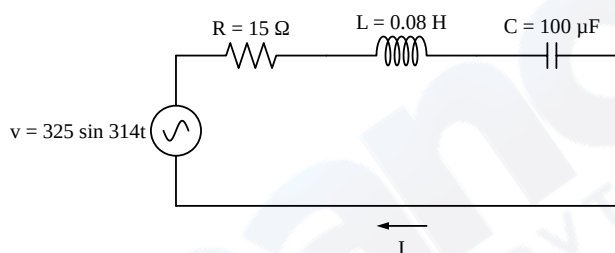
Current through 5 Ω resistor = 0.2667 A, flowing downward

(KVL equations 2 M, solution 1 M, answer 1 M)

Q.4

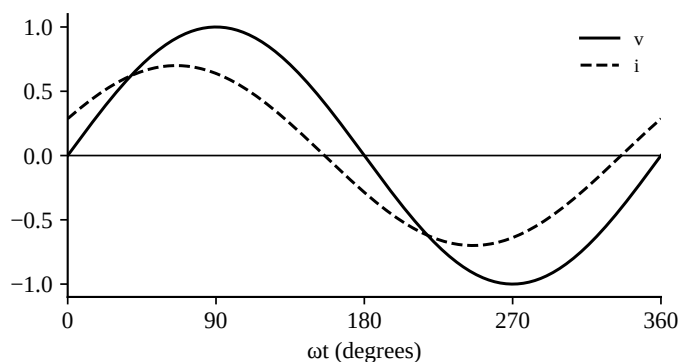
a) Series R-L-C with $v(t) = 325 \sin 314t$.

4 M



Circuit diagram

1. $\omega = 314 \text{ rad/s}$; $V_m = 325 \text{ V}$, $V_{\text{rms}} = 325/\sqrt{2} = 229.81 \text{ V}$
2. $X_L = \omega L = 314 \times 0.08 = 25.12 \Omega$
3. $X_C = 1/(\omega C) = 1/(314 \times 100 \times 10^{-6}) = 31.847 \Omega$
4. $Z = 15 + j(25.12 - 31.847) = (15 - j6.727) \Omega = \mathbf{16.439 \angle -24.16^\circ \Omega}$
5. $I_m = V_m/|Z| = 325/16.4394 = 19.7696 \text{ A}$
6. Phase angle $\phi = \tan^{-1}(X/R) = \mathbf{24.16^\circ}$; $X_C > X_L$, so current **leads** voltage
7. $\mathbf{i(t) = 19.77 \sin(314t + 24.16^\circ) \text{ A}}$
8. Power factor = $\cos 24.16^\circ = \mathbf{0.9124 \text{ leading}}$
9. Active power $P = V_{\text{rms}} I_{\text{rms}} \cos \phi = 229.81 \times 13.9792 \times 0.9124 = \mathbf{2931.27 \text{ W}}$ ($= I_{\text{rms}}^2 R$ ✓)



i leads v by 24.16°

(Z 1 M, $i(t)$ 1 M, ϕ and p.f. 1 M, power 1 M)

b) Star vs delta connection.

4 M

Point	Star (Y) connection	Delta (Δ) connection
Connection	Similar ends (star point) joined; other three ends to lines	End of one phase joined to start of next, forming a closed loop
Voltage relation	$V_L = \sqrt{3} V_{ph}$	$V_L = V_{ph}$
Current relation	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$
Neutral	Neutral point available; 3-phase 4-wire system possible	No neutral; 3-phase 3-wire only
Insulation	Phase voltage is less ($V_L/\sqrt{3}$), so less insulation needed	Phase voltage = line voltage, more insulation needed

(Any four points, 1 M each)

c) Practical current source to voltage source.

4 M

A practical current source (current I with parallel resistance R) can be replaced by an equivalent practical voltage source of emf $V = I \times R$ with the **same resistance R connected in series**.

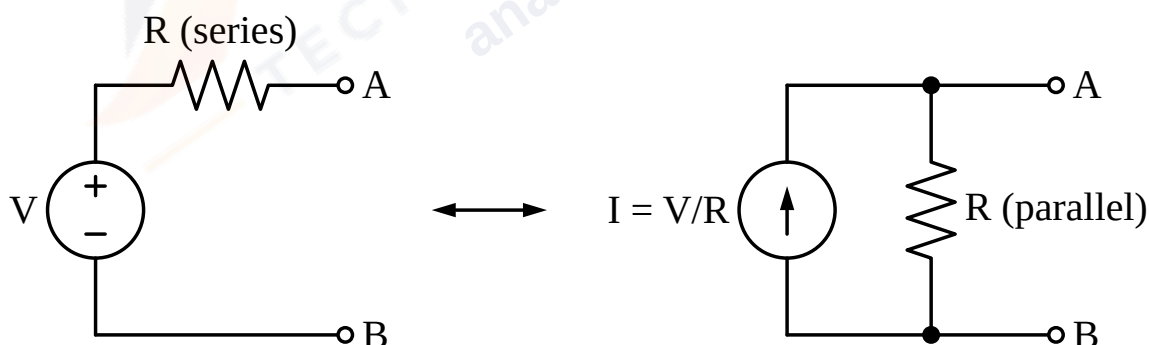


Fig.: Practical voltage source $\rightleftharpoons$ equivalent practical current source

Proof of equivalence: For load R_L : voltage-source form gives $I_L = V/(R + R_L)$; current-source form gives $I_L = I \times R/(R + R_L) = (V/R) \times R/(R + R_L) = V/(R + R_L)$ — same.

(Diagram 2 M, explanation 2 M)

d) Thevenin's theorem — statement and procedure.

4 M

Any linear, bilateral, active network having two terminals A and B can be replaced by an equivalent circuit consisting of a single voltage source V_{Th} (open-circuit voltage across A-B) in series with a single resistance R_{Th} (resistance seen from A-B with all sources replaced by their internal resistances).

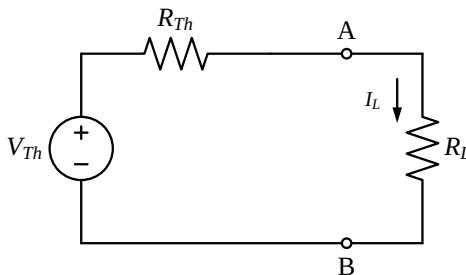


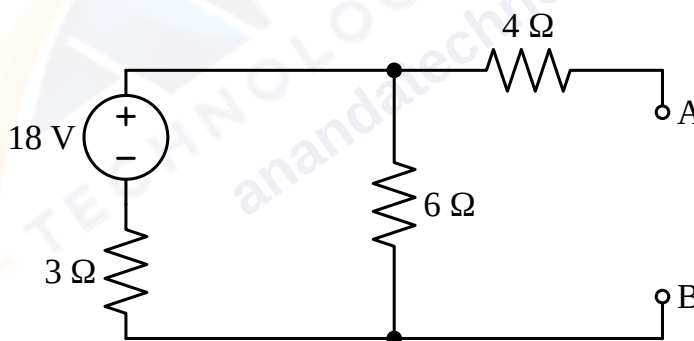
Fig.: Thevenin's equivalent circuit

1. Remove the load resistance R_L (branch through which current is required) and mark the open terminals A and B.
2. Find the open-circuit voltage across A-B using KVL / mesh / node / voltage divider: this is V_{Th} .
3. Replace all sources by their internal resistances (voltage sources short-circuited, current sources open-circuited).
4. Find the equivalent resistance seen between A and B: this is R_{Th} .
5. Draw the Thevenin equivalent circuit: V_{Th} in series with R_{Th} , and reconnect R_L across A-B.
6. Load current $I_L = V_{Th} / (R_{Th} + R_L)$.

(Statement 1 M, procedure 3 M)

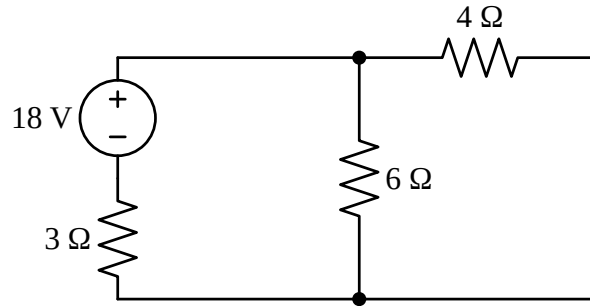
e) Norton equivalent across A-B (Fig. No. 3).

4 M



Given network

Step 1: Find I_N . Short-circuit terminals A-B:



Load terminals short-circuited

Take the bottom line as reference node (0 V). Unknown node voltage(s): V_1 . Applying KCL at each node (sum of currents leaving the node = 0):

At node V_1 : $(V_1 - 18)/3 + V_1/6 + V_1/4 = 0$

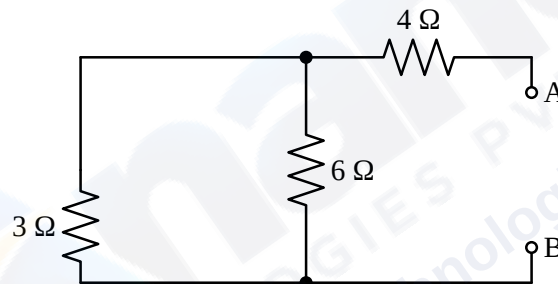
Simplifying:

$$3 V_1 = 24 \quad \dots(1)$$

Solving: $V_1 = 8 \text{ V}$

Short-circuit current (sum of currents flowing into the shorted terminals): $I_N = I_{SC} = (V_1)/4 = (8)/4 = 2 \text{ A}$

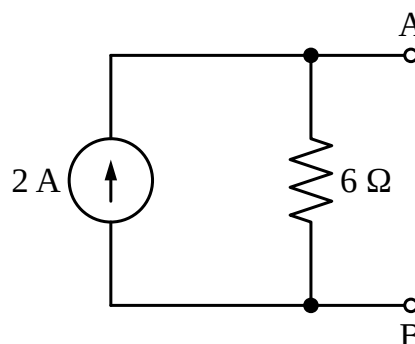
Step 2: Find R_N . Open the load terminals and replace sources by internal resistances:



Sources replaced by internal resistances

- $6 \parallel 3 = (6 \times 3)/(6 + 3) = 2 \Omega$
- $4 + 2 = 6 \Omega$ (series)
- $R_N = 6 \Omega$

Step 3: Norton equivalent circuit:



Norton equivalent across A-B

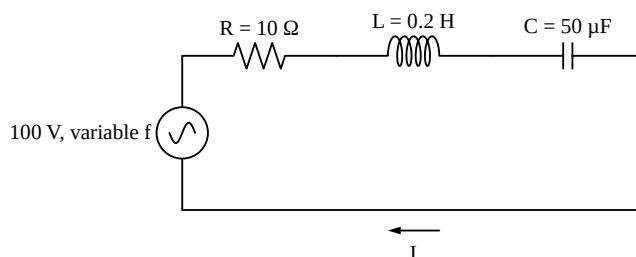
$$I_N = 2 \text{ A}, R_N = 6 \Omega$$

(I_N 2 M, R_N 1 M, equivalent circuit 1 M)

Q.5

a) Series resonance: coil 10Ω , 0.2 H with $50 \mu\text{F}$, 100 V .

6 M



Series R-L-C circuit

1. Resonant frequency $f_r = 1/(2\pi\sqrt{LC}) = 1/(2\pi\sqrt{0.2 \times 50 \times 10^{-6}}) = 50.329 \text{ Hz}$

2. At resonance $X_L = X_C = 2\pi f_r L = 63.246 \Omega$ and $Z = R = 10 \Omega$

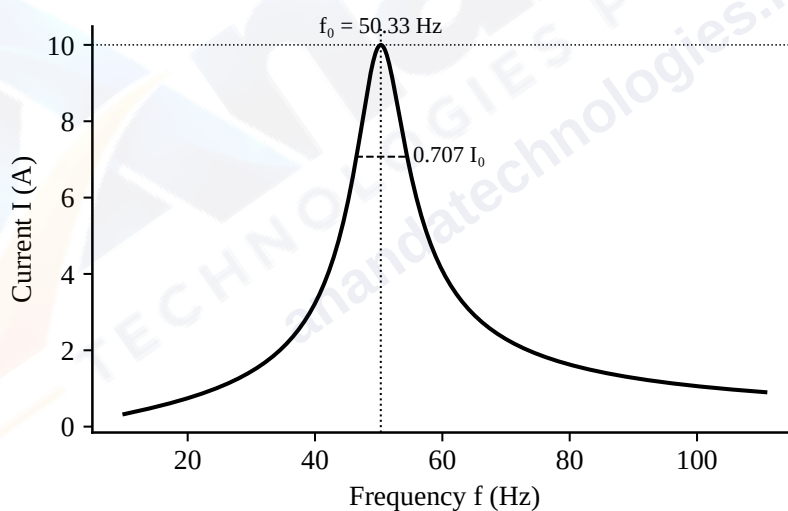
3. Current at resonance $I_0 = V/R = 100/10 = 10 \text{ A}$

4. Quality factor $Q = X_L/R = (1/R)\sqrt{L/C} = 63.246/10 = 6.3246$

5. Bandwidth $BW = R/(2\pi L) = 10/(2\pi \times 0.2) = 7.958 \text{ Hz} (= f_r/Q)$

Voltage across the coil (R and L together) $= I_0 \times \sqrt{R^2 + X_L^2} = 10 \times \sqrt{10^2 + 63.246^2} = 640.31 \text{ V}$;

voltage across capacitor $V_C = I_0 X_C = 632.46 \text{ V}$

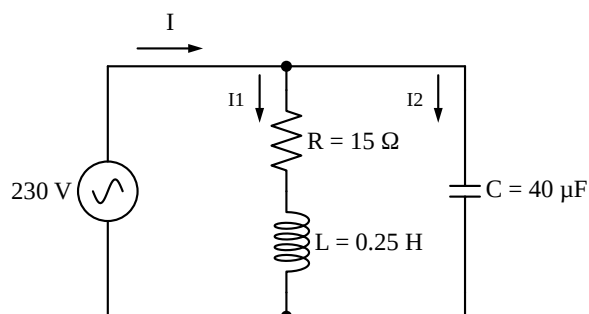


Resonance curve for the given values ($f_r = 50.33 \text{ Hz}$)

(f_r 1 M, I_0 1 M, V_{coil} and V_C 2 M, Q 1 M, BW 1 M)

$$f_r = 50.33 \text{ Hz}, I_0 = 10 \text{ A}, V_{\text{coil}} = 640.3 \text{ V}, V_C = 632.5 \text{ V}, Q = 6.325, BW = 7.958 \text{ Hz}$$

b) Parallel resonance: coil $15\ \Omega$, $0.25\ \text{H}$ || $40\ \mu\text{F}$, $230\ \text{V}$.

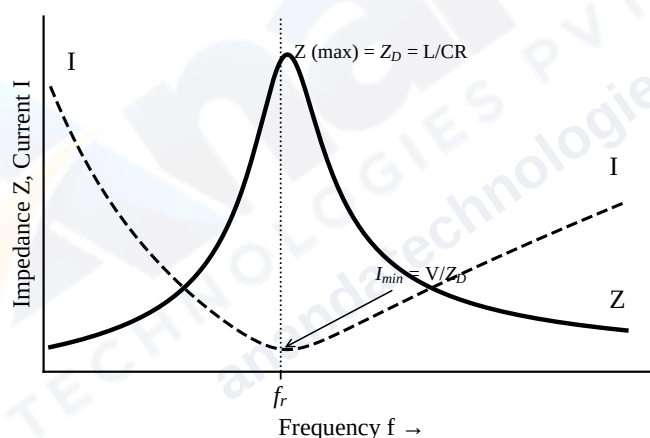


Coil in parallel with capacitor

1. Resonant frequency $f_r = (1/2\pi)\sqrt{(1/LC - R^2/L^2)} = (1/2\pi)\sqrt{(1/(0.25 \times 40 \times 10^{-6}) - 15^2/0.25^2)} = \mathbf{49.415\ \text{Hz}}$
2. Dynamic impedance $Z_D = L/(CR) = 0.25/(40 \times 10^{-6} \times 15) = \mathbf{416.67\ \Omega}$
3. Current at resonance (line current) $I = V/Z_D = 230/416.67 = \mathbf{0.552\ \text{A}}$ (minimum, in phase with V)
4. Q-factor of coil $Q = \omega_r L/R = 2\pi \times 49.415 \times 0.25/15 = \mathbf{5.1747}$
5. Power consumed $P = VI$ (p.f. = 1) = $230 \times 0.552 = \mathbf{126.96\ \text{W}}$ ($= I_{\text{coil}}^2 R = 2.9093^2 \times 15 = 126.96\ \text{W}$)

Capacitor current at resonance $I_C = V \times 2\pi f_r C = 2.8564\ \text{A}$; current magnification $I_C/I = 5.175 \approx Q$.

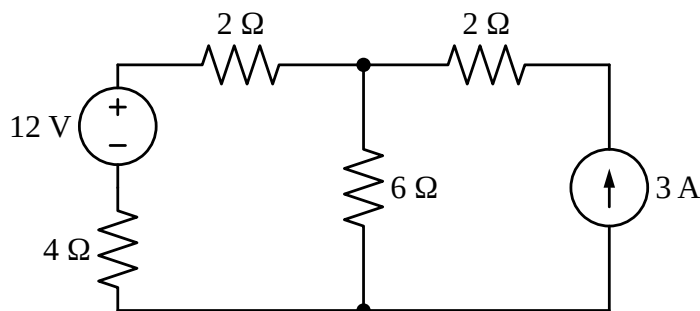
(f_r 2 M, Z_D 1 M, I 1 M, Q 1 M, P 1 M)



Resonance curve (nature)

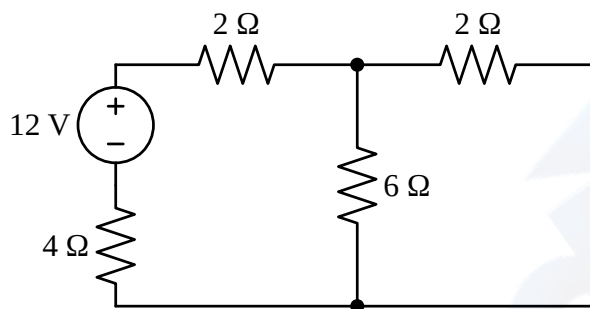
$f_r = 49.41\ \text{Hz}$, $Z_D = 416.67\ \Omega$, $I = 0.552\ \text{A}$, $Q = 5.175$, $P = 126.96\ \text{W}$

c) Superposition — current in 6 Ω (Fig. No. 4).



Given network

Step 1: 12 V source acting alone (other current source open-circuited):

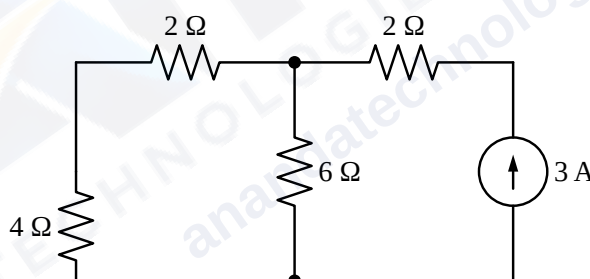


12 V source acting alone

- $2 + 6 = 8 \Omega$ (series)
- Current supplied by 12 V source = $12/(4 + 8) = 1 \text{ A}$

I_1' in 6 Ω = 1 A downward

Step 2: 3 A current source acting alone (other voltage source short-circuited):



3 A current source acting alone

- $2 + 4 = 6 \Omega$ (series)
- $6 \parallel 6 = (6 \times 6)/(6 + 6) = 3 \Omega$
- $2 + 3 = 5 \Omega$ (series)
- Source current entering the node = 3 A
- Current in the required branch = $3 \times 6/(6 + 6) = 1.5 \text{ A}$

I_2' in 6 Ω = 1.5 A downward

Total current (algebraic sum, taking downward as positive):

Source acting	Current in $6\ \Omega$
12 V source	1 A downward
3 A current source	1.5 A downward
Total	2.5 A downward

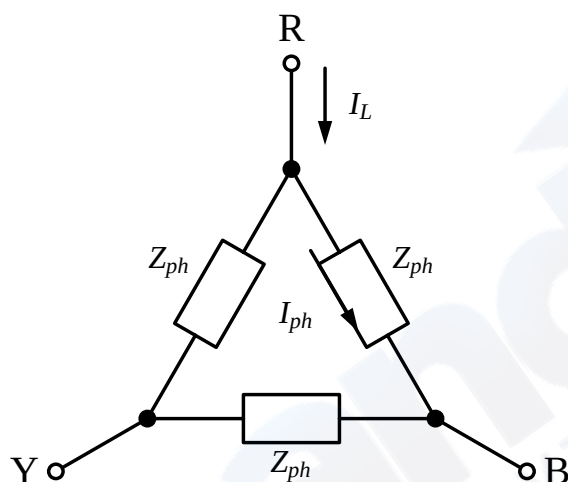
Current through $6\ \Omega = 2.5\ \text{A}$ (downward)

(Each source acting alone 2 M each, total 2 M)

Q.6

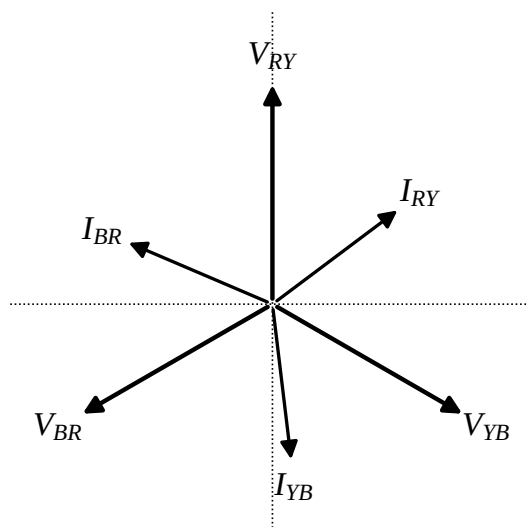
a) Delta load $(6 + j8)\ \Omega$ per phase, 415 V.

6 M



Balanced delta connected load, $Z_{ph} = (6 + j8)\ \Omega$

- Delta connection: $V_{ph} = V_L = 415\ \text{V}$
- Impedance per phase $Z_{ph} = (6 + j8)\ \Omega$; $|Z_{ph}| = \sqrt{6^2 + 8^2} = 10\ \Omega$
- Phase current $I_{ph} = V_{ph}/Z_{ph} = 415/10 = 41.5\ \text{A}$
- Line current $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 41.5 = 71.8801\ \text{A}$ (delta)
- Power factor $\cos \phi = R_{ph}/Z_{ph} = 6/10 = 0.6$ lagging
- Total active power $P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 415 \times 71.8801 \times 0.6 = 31000.5\ \text{W}$ (check:
 $3I_{ph}^2 R_{ph} = 31000.5\ \text{W}$)
- Total reactive power $Q = \sqrt{3} V_L I_L \sin \phi = 41334\ \text{VAR}$
- Total apparent power $S = \sqrt{3} V_L I_L = 51667.5\ \text{VA}$



Phasor diagram: phase currents lag phase voltages by 53.13° ($V_L = V_{ph}$)

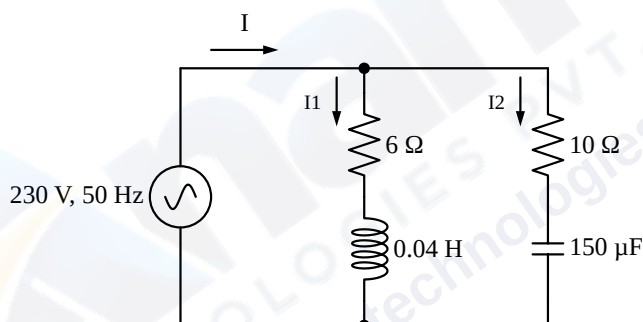
(I_{ph} 1 M, I_L 1 M, p.f. 1 M, powers 2 M, phasor diagram 1 M)

$I_{ph} = 41.5$ A, $I_L = 71.88$ A, p.f. = 0.6 lagging, $P = 31$ kW, $Q = 41.334$ kVAR, $S = 51.667$ kVA

b) Coil (6Ω , 0.04 H) $\parallel$ ($10 \Omega + 150 \mu\text{F}$), 230 V, 50 Hz.

6 M

1. Coil branch: $X_L = 2\pi \times 50 \times 0.04 = 12.566 \Omega \Rightarrow Z_1 = (6 + j12.566) \Omega$
2. R-C branch: $X_C = 1/(2\pi \times 50 \times 150 \times 10^{-6}) = 21.221 \Omega \Rightarrow Z_2 = (10 - j21.221) \Omega$

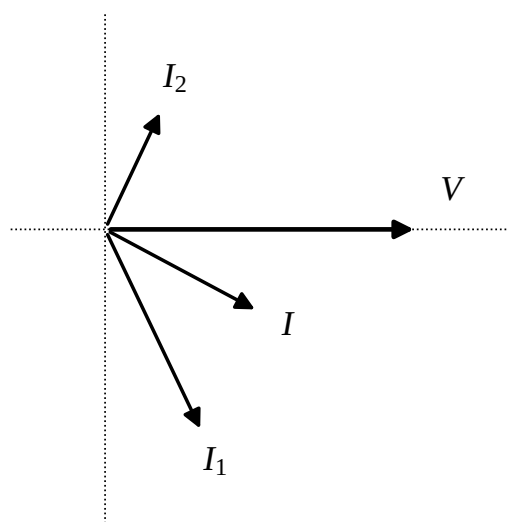


Circuit diagram

1. $Z_1 = (6 + j12.566) \Omega = 13.925 \angle 64.48^\circ \Omega$
2. $Z_2 = (10 - j21.221) \Omega = 23.459 \angle -64.77^\circ \Omega$
3. $Z_1 + Z_2 = (16 - j8.654) = 18.191 \angle -28.41^\circ$
4. Equivalent impedance $Z = Z_1 Z_2 / (Z_1 + Z_2) = (13.925 \angle 64.48^\circ)(23.459 \angle -64.77^\circ) / (18.191 \angle -28.41^\circ) = 17.958 \angle 28.12^\circ \Omega = (15.839 + j8.463) \Omega$
5. Branch current $I_1 = V/Z_1 = 230 \angle 0^\circ / 13.925 \angle 64.48^\circ = 16.517 \angle -64.48^\circ$ A = (7.117 - j14.905) A
6. Branch current $I_2 = V/Z_2 = 230 \angle 0^\circ / 23.459 \angle -64.77^\circ = 9.804 \angle 64.77^\circ$ A = (4.179 + j8.869) A
7. Total current $I = I_1 + I_2 = (11.296 - j6.036)$ A = **$12.807 \angle -28.12^\circ$ A**
1. p.f. of branch 1 = $\cos \phi_1 = R_1/|Z_1| = 6/13.9253 = 0.4309$ lagging

2. p.f. of branch 2 = $\cos \phi_2 = R_2/|Z_2| = 10/23.4588 = \mathbf{0.4263 \text{ leading}}$

3. Overall p.f. = $\cos(\text{angle between } V \text{ and } I) = \cos 28.12^\circ = \mathbf{0.882 \text{ lagging}}$



Phasor diagram (V as reference)

(Branch currents 2 M, total current/p.f. 2 M, power/phasor diagram 2 M)

$I_1 = 16.517 \angle -64.48^\circ \text{ A}$, $I_2 = 9.804 \angle 64.77^\circ \text{ A}$, $I = 12.807 \angle -28.12^\circ \text{ A}$, p.f. = **0.882 lagging**

c) Maximum power transfer — proof and numerical (Fig. No. 5).

6 M

A resistive load connected to a DC network receives **maximum power** when the load resistance is equal to the Thevenin's (internal) resistance of the network as seen from the load terminals: $R_L = R_{Th}$.

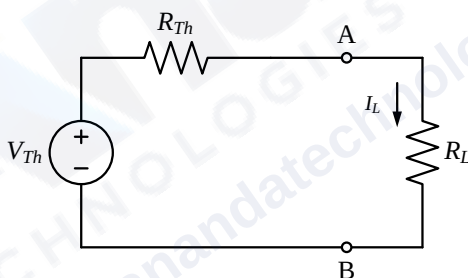
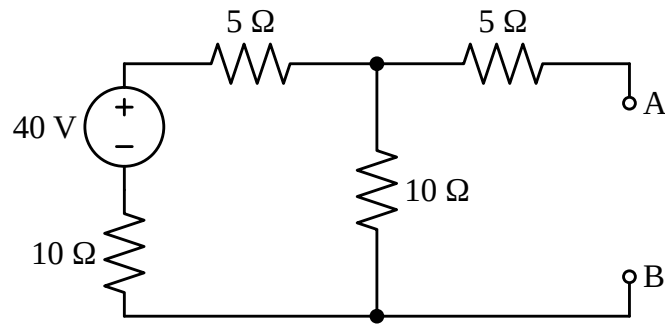


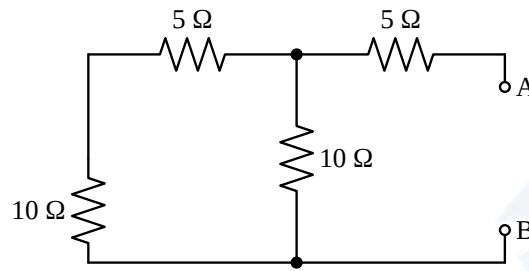
Fig.: Load R_L connected to Thevenin equivalent of the network

- Load current $I_L = V_{Th}/(R_{Th} + R_L)$; power in load $P_L = I_L^2 R_L = V_{Th}^2 R_L / (R_{Th} + R_L)^2$.
- For maximum power, $dP_L/dR_L = 0$: $V_{Th}^2 [(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)] / (R_{Th} + R_L)^4 = 0$
- $(R_{Th} + R_L) - 2R_L = 0 \Rightarrow R_L = R_{Th}$
- Substituting: $P_{max} = V_{Th}^2 R_L / (2R_L)^2 = V_{Th}^2 / (4R_L) = V_{Th}^2 / (4R_{Th})$
- Efficiency at maximum power transfer = 50 % (half the power is lost in R_{Th}).



Given network (variable load across A-B)

Step 1: R_{Th} (sources replaced by internal resistances):



Sources killed

- $5 + 10 = 15 \Omega$ (series)
- $10 \parallel 15 = (10 \times 15)/(10 + 15) = 6 \Omega$
- $5 + 6 = 11 \Omega$ (series)
- $R_{Th} = 11 \Omega$

Step 2: V_{Th} (open-circuit voltage across A-B):

Take the bottom line as reference node (0 V). Unknown node voltage(s): V_1, V_2, V_3 . Applying KCL at each node (sum of currents leaving the node = 0):

At node V_1 : $(V_1 - 40)/10 + (V_1 - V_2)/5 = 0$

At node V_2 : $V_2/10 + (V_2 - V_1)/5 + (V_2 - V_3)/5 = 0$

At node V_3 : $(V_3 - V_2)/5 = 0$

Simplifying:

$$3V_1 - 2V_2 = 40 \quad \dots(1)$$

$$-2V_1 + 5V_2 - 2V_3 = 0 \quad \dots(2)$$

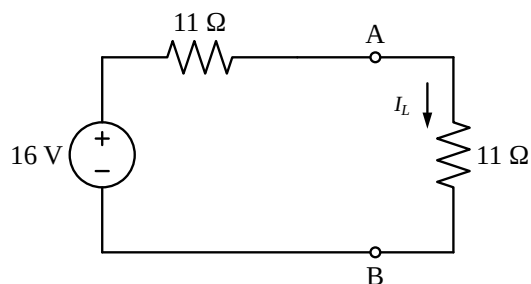
$$-V_2 + V_3 = 0 \quad \dots(3)$$

Solving: $V_1 = 24 \text{ V}$, $V_2 = 16 \text{ V}$, $V_3 = 16 \text{ V}$

V_{Th} = voltage of the open-circuited terminal node = 16 V

Step 3: For maximum power transfer, $R_L = R_{Th} = 11 \Omega$

$$P_{\max} = V_{Th}^2/(4R_L) = 16^2/(4 \times 11) = 5.8182 \text{ W}$$



Thevenin equivalent with $R_L = R_{Th}$

$$R_L = 11 \, \Omega, P_{\max} = 5.8182 \, \text{W}$$

(Proof 2 M, R_{Th} 1 M, V_{Th} 2 M, $P_{\max}$ 1 M)

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