Theory of Structure (315313)

MSBTE K-Scheme · Civil Engineering (CE) Semester 5 · Model Question Paper — Set B · Model Answers

Sign conventions used: sagging BM positive (+), hogging BM negative (−) in diagrams; support moments in three-moment equations are written as hogging magnitudes; in moment distribution, clockwise end moments are positive. Units: kN, m unless stated.

Q.1 Attempt any FIVE of the following: (10 M)

a) Relation between BM, slope and deflection — 2 M

If y is the deflection at a section at distance x, then for small deflections:

- Deflection = **y**
- Slope = **dy/dx** = θ
- Bending moment M = **EI d^2y/dx^2**
- Shear force F = **EI d^3y/dx^3**
- Rate of loading w = **EI d^4y/dx^4**

Also from the flexure formula M/I = E/R, so 1/R = M/EI = d^2y/dx^2 (R = radius of curvature).

(Any two correct relations, 1 M each)

b) Carry-over moment and carry-over factor — 2 M

Carry-over moment: When a moment M is applied at one end (near end) of a member whose far end is fixed, a moment of the same sense and **half its magnitude (M/2)** is developed at the far end. This moment is called the carry-over moment.

Carry-over factor: The ratio of the carry-over moment at the far end to the applied moment at the near end; **C.O.F. = ½** for a prismatic member with far end fixed (0 if the far end is hinged).

(Each 1 M)

c) Perfect and imperfect frames — 2 M

Perfect frame: A frame which has just sufficient number of members to remain in equilibrium without any distortion when loaded. It satisfies **m = 2j − 3** (m = members, j = joints). Example: a triangle (m = 3, j = 3).

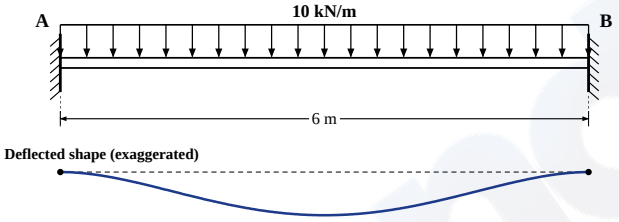
Imperfect frame: A frame which does not satisfy m = 2j − 3. It is (i) **deficient** if m < 2j − 3 (unstable, collapses) or (ii) **redundant** if m > 2j − 3 (statically indeterminate).

(a) Perfect: m = 3, j = 3, 2j−3 = 3 (b) Deficient: m = 4 < 2j−3 = 5 (c) Redundant: m = 6 > 2j−3 = 5

(Each definition 1 M)

d) Fixed beam 2 M

Fixed beam: A beam whose **both ends are rigidly fixed** (built-in) so that at the supports the **slope and deflection are zero** is called a fixed beam (also built-in or encastré beam). Fixing moments (hogging) develop at the supports. Example: RCC beam cast monolithic with columns.



Fixed beam: slope = 0 and deflection = 0 at A and B

(Definition 2 M)

e) Flexural rigidity and stiffness 2 M

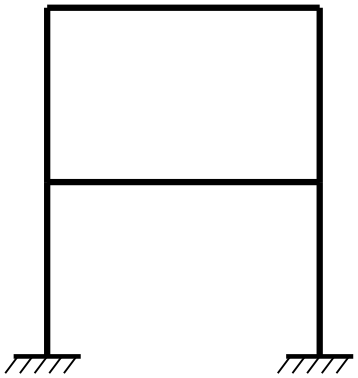
Flexural rigidity: The product **EI** of modulus of elasticity E and moment of inertia I of the section is called flexural rigidity. It is the resistance of the beam to bending (unit kN·m² or N·mm²).

Stiffness of beam: The **load required to cause unit deflection** (k = W / y). Greater EI and shorter span give a stiffer beam. E.g. for SS beam with central load, stiffness = 48EI/L³.

(Each 1 M)

f) Portal frame — one bay, two storeys 2 M

A portal frame is a rigid-jointed frame of columns and beams. A symmetrical frame with one bay and 2 storeys has equal columns on both sides, equal storey heights and identical supports:



Symmetrical portal frame — 1 bay, 2 storey (equal bay width, equal storey height, fixed bases)

(Correct sketch 2 M)

g) Boundary conditions at fixed end of cantilever 2 M

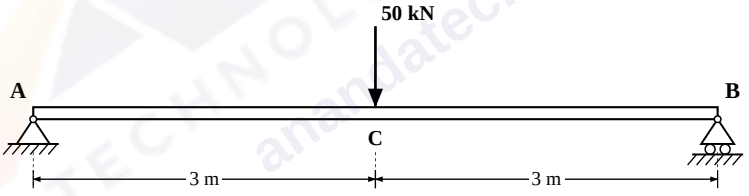
For a cantilever fixed at A and free at B (origin at free end, fixed end at x = L):

- At the **fixed end**: deflection **y = 0** and slope **dy/dx = 0**.
- At the free end: bending moment = 0 (and shear = 0 if no point load); slope and deflection are maximum.

(Each condition 1 M)

Q.2 Attempt any THREE of the following: (12 M)

a) SS beam with central point load — standard formulae 4 M



Simply supported beam with central point load

Given: L = 6 m, W = 50 kN, E = 200,000 N/mm², I = 1.2×10^8 mm⁴

EI = $200{,}000 \times 1.2 \times 10^8 = 2.4 \times 10^{13}$ N·mm² = **24000 kN·m²**

Standard formulae (SS beam, central point load):

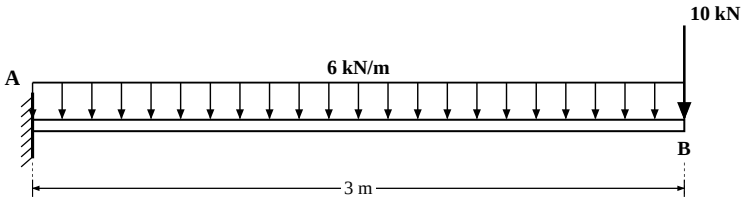
$\theta_{max} = \theta_A = \theta_B = WL^2 / 16EI$; $y_{max} = y_C = WL^3 / 48EI$

$\theta_{max} = 50 \times 6^2 / (16 \times 24000) =$ **0.00469 rad** = 0.269°

$y_{max} = 50 \times 6^3 / (48 \times 24000) = 0.009375$ m = **9.38 mm** (at centre)

(Formulae 1 M, EI 1 M, slope 1 M, deflection 1 M)

b) Cantilever with UDL and end load 4 M



Cantilever AB (A fixed, B free)

Given: L = 3 m, w = 6 kN/m, W = 10 kN at free end, E = 200,000 N/mm²

EI = 200,000 × 6 × 10^7 = 1.2 × 10^{13} N·mm² = **12000 kN·m²**

Load case	Slope at B	θ (rad)	Deflection at B	y (mm)
Point load W at free end	$WL^2/2EI$	0.00375	$WL^3/3EI$	7.50
UDL w on whole span	$wL^3/6EI$	0.00225	$wL^4/8EI$	5.06

By the principle of superposition, the total slope and deflection are the sum of the two cases:

Slope at free end θ_B = **0.00600 rad** (0.344°)

Deflection at free end y_B = **12.56 mm** (downward)

(I and EI 1 M, formulae 1 M, slope 1 M, deflection 1 M)

c) Fixed beam with UDL — first principles 4 M

First principle (area method): For a fixed beam, since the slope at both ends is zero, (i) the area of the free BMD = area of the fixed-end moment diagram, and (ii) their centroids lie on the same vertical line.

For a UDL w over span L, the free BMD is a parabola with maximum ordinate $wL^2/8$.

Area of free BMD = (2/3) × L × $wL^2/8$ = **$wL^3/12$**. Due to symmetry $M_A = M_B = M$, so area of fixed-moment diagram = M × L.

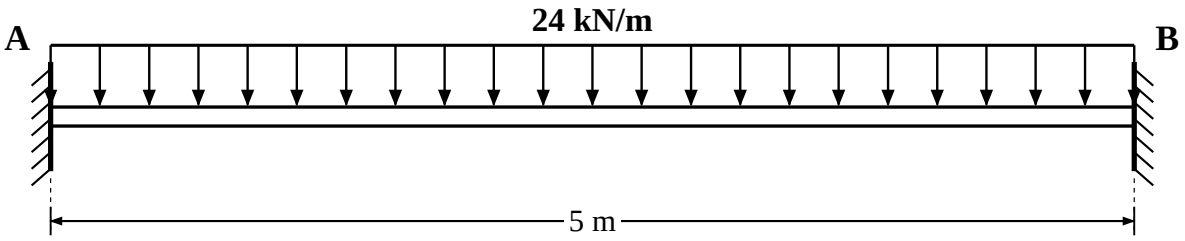
M × L = $wL^3/12$ → **$M_A = M_B = wL^2/12$** (hogging).

Given: L = 5 m, w = 24 kN/m.

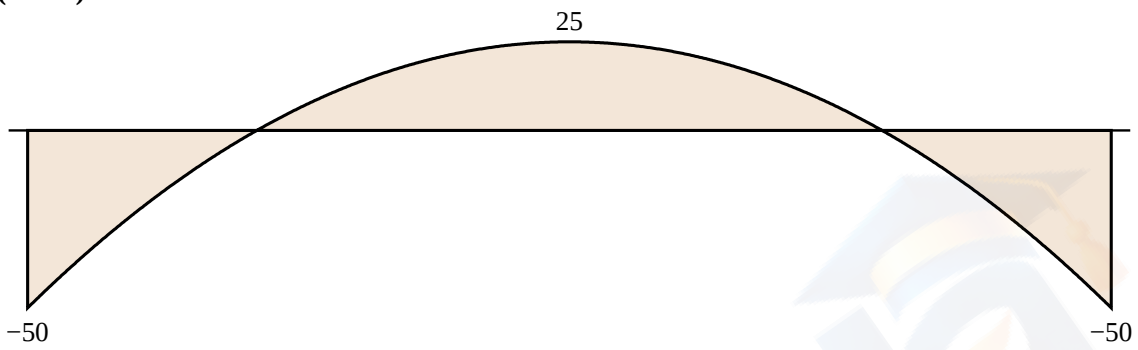
Fixed end moments $M_A = M_B = 24 \times 5^2/12$ = **50 kN·m (hogging)**

Free BM at centre = $wL^2/8$ = 75 kN·m. Net BM at centre = 75 − 50 = **25 kN·m (sagging)**.
Reactions $R_A = R_B = wL/2$ = 60 kN.

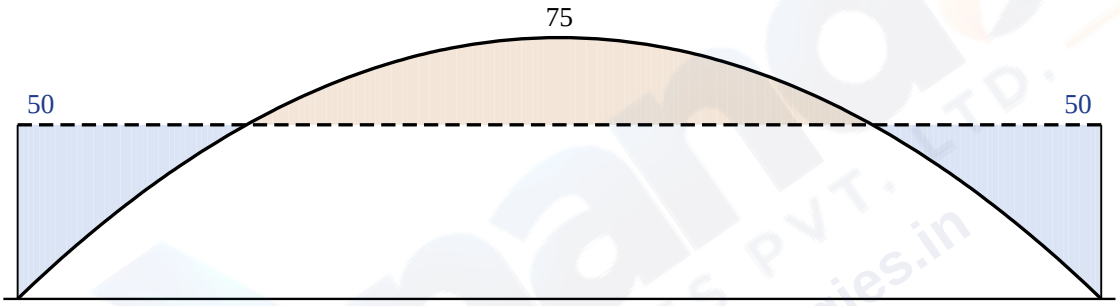
Points of contraflexure: x = 0.2113L = **1.06 m** from each end.



BMD (kN·m)



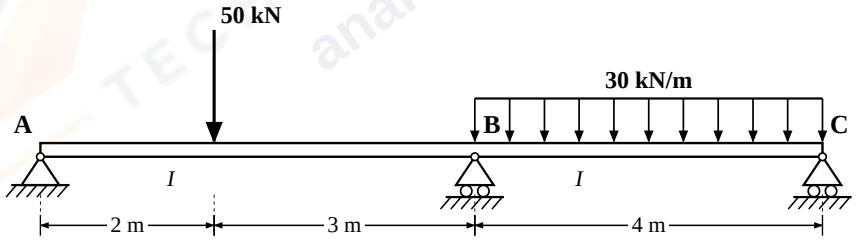
BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



Fixed beam with UDL — net BMD and BMD by superposition

(First principle 2 M, FEM 1 M, BMD 1 M)

d) Three moment theorem — support moment at B 4 M



Step 1 — Free bending moments and 6Ax̄/L terms (each span treated as simply supported). For UDL: 6Ax̄/L = wL³/4; for point load W at distance a from the left end: 6Ax̄/L = Wab(L + a)/L about the left end and Wab(L + b)/L about the right end.

Span	L (m)	Max free BM (kN·m)	Area A	x̄ from left (m)	x̄ from right (m)	$6A\bar{x}_{left}/L$	$6A\bar{x}_{right}/L$
AB	5	Wab/L = 50×2×3/5 = 60	150	2.33	2.67	420	480

BC	4	$wL^2/8 = 30\times4^2/8 = 60$	160	2	2	480	480

Step 2 — Clapeyron's equations (support moments M taken hogging positive):

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = 6A_1\bar{x}_1/L_1 + 6A_2\bar{x}_2/L_2$$

($\bar{x}_1$ measured from the left support of the left span, $\bar{x}_2$ from the right support of the right span.)

End A is simply supported, so $M_A = 0$. End C is simply supported, so $M_C = 0$.

For supports **A, B, C**: $18\ M_B = 900$ … (1)

Solving: $\mathbf{M_B = 50\ kN\cdot m}$ (hogging).

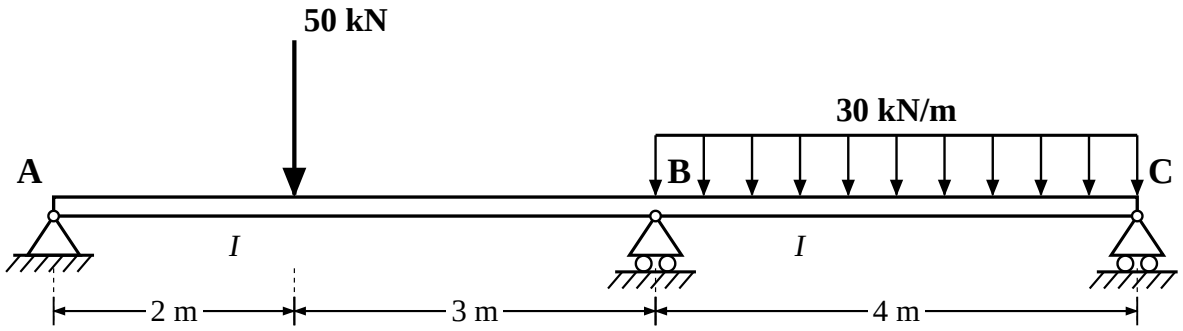
Step 3 — Reactions, SFD and BMD

Support reactions (span-wise: free reaction ± $(M_{left} - M_{right})/L$):

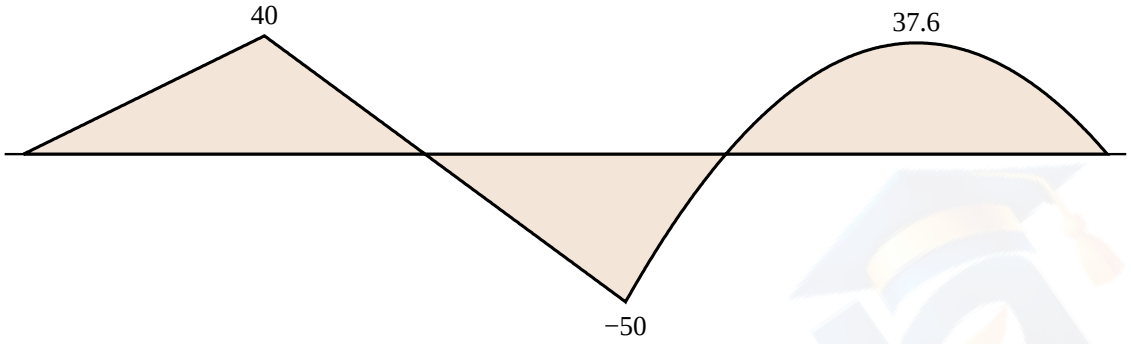
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (5 m)	30 / 20	$(0 - 50)/5 = -10$	20	30
BC (4 m)	60 / 60	$(50 - 0)/4 = 12.5$	72.5	47.5

$\mathbf{R_A = 20\ kN}$; $\mathbf{R_B = 102.5\ kN}$; $\mathbf{R_C = 47.5\ kN}$ ($\Sigma R = 170$ kN = total load ✓)

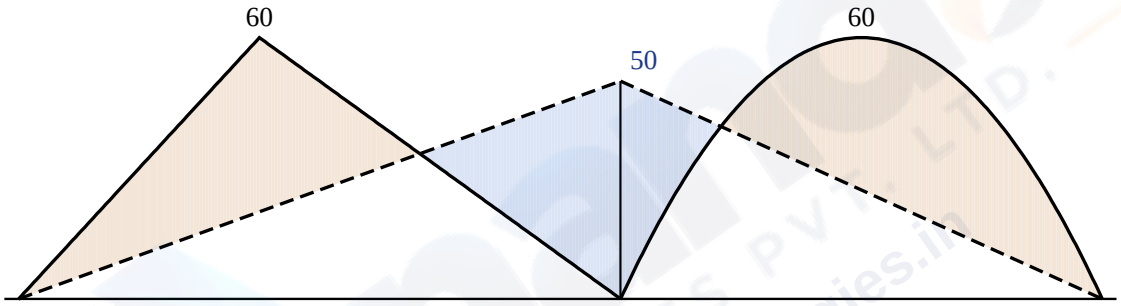
Maximum sagging moments — span AB: **40 kN·m** at 2 m from A; span BC: **37.6 kN·m** at 2.42 m from B.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



BMD (net) and BMD by superposition (+ sagging, − hogging)

✔ Check: support moments and reactions agree with an independent stiffness-matrix (and moment distribution) solution. *(6Aẋ/L terms 2 M, equation and M_B 2 M)*

Q.3 Attempt any THREE of the following: (12 M)

a) Clapeyron's theorem of three moments

4 M

Statement: For a continuous beam with two consecutive spans L_1 and L_2 supported at A, B and C (at the same level), the support moments M_A, M_B and M_C are related as

$$\mathbf{M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6A_1\bar{x}_1/L_1 + 6A_2\bar{x}_2/L_2}$$

where A_1, A_2 = areas of the free BMDs of spans AB and BC (each treated as simply supported); $\bar{x}_1$ = distance of centroid of A_1 from the left end A; $\bar{x}_2$ = distance of centroid of A_2 from the right end C. Support moments are hogging. If the M.I. differs, each L is replaced by L/I and each 6Aẋ/L by 6Aẋ/(LI).

Explanation / use: The equation is written for every set of three consecutive supports; the simultaneous equations give the unknown support moments. For a simply supported end the

moment is zero; for a fixed end an imaginary span of zero length is added. Standard values: UDL → $6A\bar{x}/L = wL^3/4$; central point load → $6A\bar{x}/L = 3WL^2/8$.

Free BMDs (areas A_1, A_2) and support moment line (dashed)

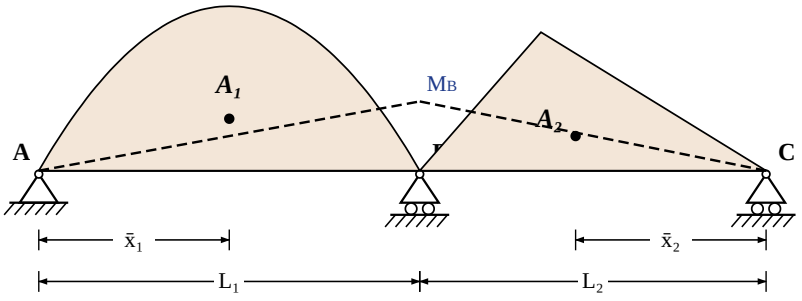
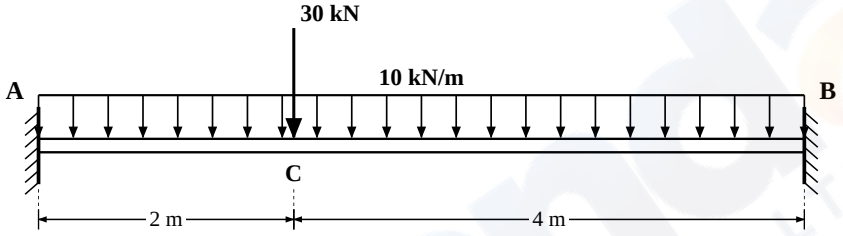


Fig.: Two consecutive spans for Clapeyron's theorem of three moments

(Statement 2 M, explanation with sketch 2 M)

b) Fixed beam — FEM and BMD 4 M



Step 1 — Fixed end moments (hogging, by standard formulae):

Load	M_A (kN·m)	M_B (kN·m)
UDL 10 kN/m on whole span	$wL^2/12 = 10\times6^2/12 = 30$	$wL^2/12 = 30$
Point load 30 kN at 2 m from A (a = 2, b = 4)	$Wab^2/L^2 = 30\times2\times4^2/6^2 = 26.67$	$Wa^2b/L^2 = 30\times2^2\times4/6^2 = 13.33$
Total	**56.67**	**43.33**

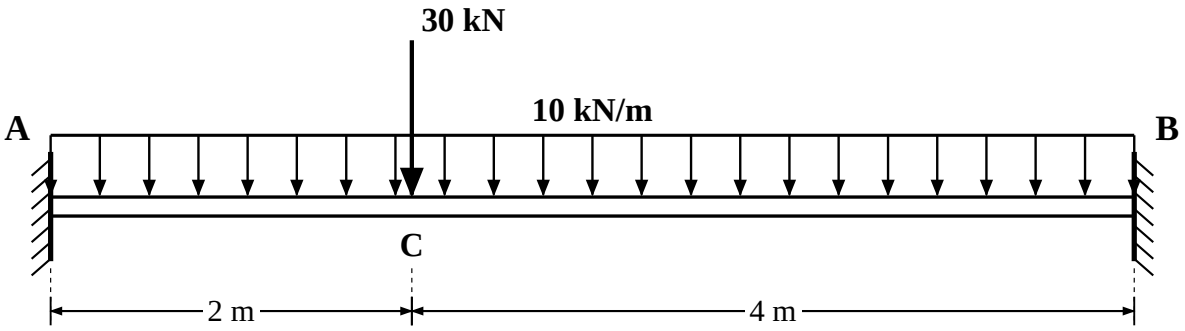
Fixed end moments: $\mathbf{M_A = -56.67\ kN\cdot m}$ and $\mathbf{M_B = -43.33\ kN\cdot m}$ (negative = hogging).

Step 2 — Reactions: R_A = (free reaction) + $(M_A - M_B)/L = 50 + (56.67 - 43.33)/6 =$ **52.22 kN**; $R_B = 40 - (56.67 - 43.33)/6 =$ **37.78 kN**

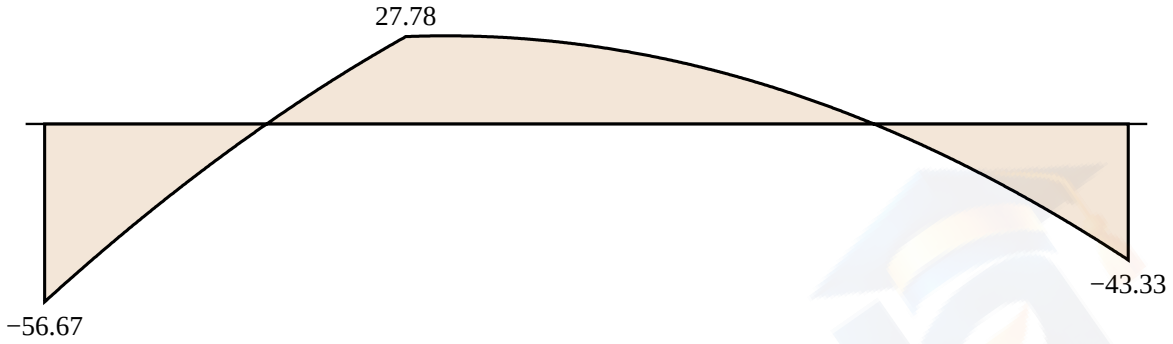
Step 3 — Bending moments (net BM = free BM of SS beam − fixed end moment line):

Section (from A)	Free BM (kN·m)	Fixed-moment line (kN·m)	Net BM (kN·m)
A (0 m)	0	56.67	**−56.67**
2 m	80	52.22	**27.78**
3 m	75	50	**25**
B (6 m)	0	43.33	**−43.33**

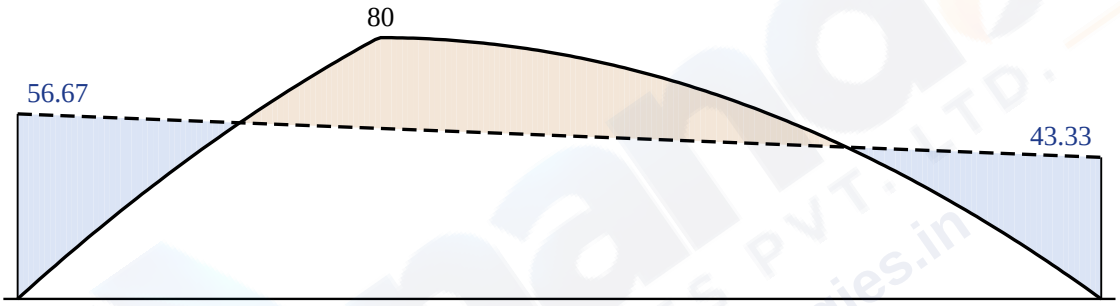
Points of contraflexure (BM = 0): **1.23 m**, **4.59 m** from A.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



Deflected shape (exaggerated)

Fixed beam — net BMD, BMD by superposition and deflected shape

✔ Check: stiffness-matrix analysis gives the same end moments (56.67, 43.33 kN·m).

(FEMs 2 M, BM values 1 M, BMD 1 M)

c) Distribution factors at joint O

4 M

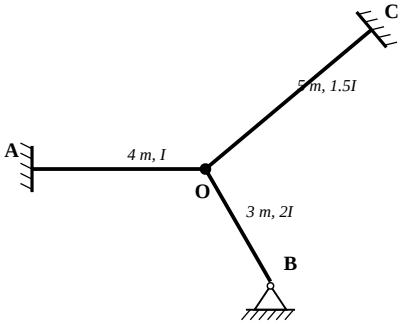


Fig.: Members meeting at rigid joint O

Stiffness of a member: **k = 4EI/L** when the far end is fixed and **k = 3EI/L** when the far end is hinged / simply supported. Distribution factor = k / Σk at joint O.

Member	Length	M.I.	Far end	Stiffness k	D.F.
OA	4 m	I	Fixed	4EI/4 = 1EI	1/4.2 = **0.238**
OB	3 m	2I	Hinged	3E2I/3 = 2EI	2/4.2 = **0.476**
OC	5 m	1.5I	Fixed	4E1.5I/5 = 1.2EI	1.2/4.2 = **0.286**

Σk = 4.2EI; ΣDF = 1 ✓

(Stiffness 2 M, distribution factors 2 M)

d) Assumptions in analysis of trusses 4 M

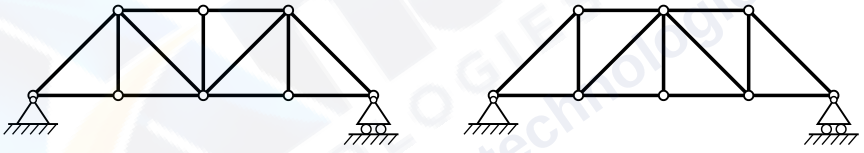
1. All members are **pin-jointed** (hinged) at their ends; joints are frictionless.
2. **Loads act only at the joints** (nodes).
3. The **self-weight of members is neglected**.
4. All members are **straight** and carry only axial force (tension or compression) — no bending.
5. The frame is a **perfect frame** (m = 2j − 3) and all members lie in one plane.

(Any four, 1 M each)

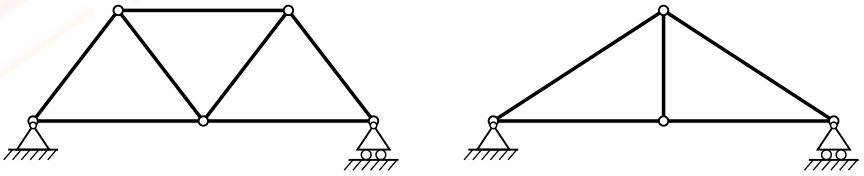
Q.4 Attempt any THREE of the following: (12 M)

a) Types of trusses — sketches 4 M

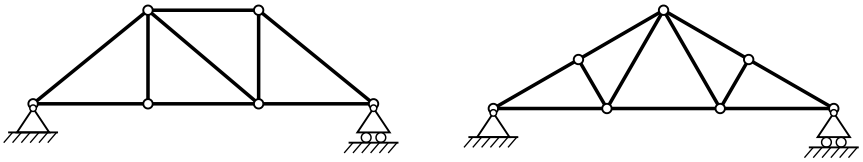
Commonly used trusses:



Pratt truss *Howe truss*



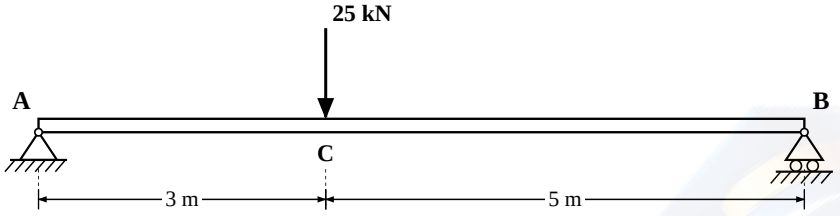
Warren truss *King post truss*



Queen post truss *Fink (French) truss*

(Any four neat sketches, 1 M each)

b) Macaulay's method — deflection under load 4 M



Step 1 — Reactions: $\Sigma M_A = 0$: $R_B \times 8 = 25 \times 3 \rightarrow$ **R_B = 9.38 kN**; R_A = (total load) − R_B = **15.62 kN**

Step 2 — Macaulay's equation: Take A as origin and a section X–X at distance x from A in the last segment. (Terms after the vertical bar apply only when the bracket is positive.)

$EI\ d^2y/dx^2 = M_x = 15.625x \mid -25[x-3]$

$EI\ dy/dx = 7.812x^2 \mid -12.5[x-3]^2 + C_1 \quad \ldots (1)$

$EI\ y = 2.604x^3 \mid -4.167[x-3]^3 + C_1x + C_2 \quad \ldots (2)$

Step 3 — Boundary conditions: At x = 0, y = 0 → (first segment of eq. 2) **C_2 = 0**. At x = 8 m, y = 0 → (all terms of eq. 2) **C_1 = −101.562**

Step 4 — Results:

- Deflection at C (x = 3 m): EI y = −234.375 → **y_C = 234.37/EI** (downward)
- Deflection at mid-span (x = 4 m): EI y = −243.75 → **$y_{mid\text{-}span}$ = 243.75/EI** (downward)

✔ Check: values agree with an independent stiffness-matrix analysis of the same beam.

(Reactions and equation 1 M, constants 1 M, two deflections 2 M)

c) Radius of curvature, stiffness, elastic curve 4 M

Radius of curvature (R): When a beam bends, its neutral axis takes the shape of a circular arc. The radius of this arc is called the radius of curvature R. From bending theory M/I = E/R → **R = EI/M**. Greater the BM, smaller R (more curvature); at a point of zero BM, R is infinite.

Stiffness of beam: The resistance offered by a beam against deflection. It is measured as the **load required to produce unit deflection**, k = W/y. It depends on E, I (i.e. EI), span and support conditions. For a SS beam with central load: $y = WL^3/48EI$ → stiffness = **$48EI/L^3$**.

Elastic curve: The deflected shape of the neutral axis is called the elastic curve. For a SS beam with central point load it is a symmetrical sagging curve with maximum deflection $WL^3/48EI$ at the

centre and maximum slope $WL^2/16EI$ at the supports.

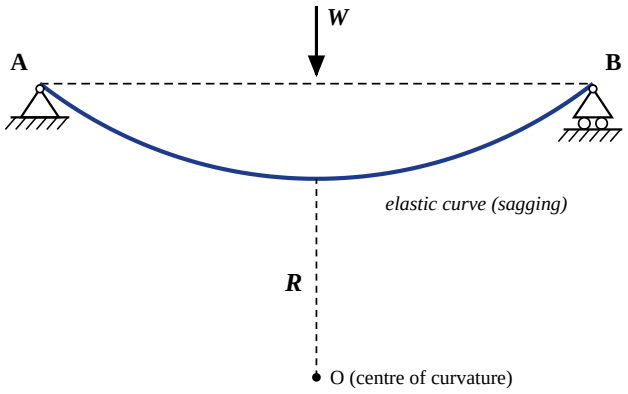


Fig.: Elastic curve of a SS beam with central point load and radius of curvature R

(Radius of curvature 1½ M, stiffness 1½ M, sketch 1 M)

d) Procedure of moment distribution method 4 M

1. **Fixed end moments:** Treat every span as fixed at both ends and calculate FEMs (clockwise +, anticlockwise −).
2. **Stiffness:** Find the relative stiffness of each member at each joint: k = I/L (far end fixed/continuous) or (3/4)I/L (far end hinged).
3. **Distribution factors:** DF = k/Σk at each joint (DF = 0 at a fixed end, 1 at a free simple end).
4. **Release simple ends:** balance the FEM at simply supported ends to zero and carry over half to the other end.
5. **Balance:** At each intermediate joint, find the unbalanced moment (algebraic sum of moments), and distribute it with **opposite sign** in proportion to the DFs.
6. **Carry over:** Carry over half of each distributed moment to the far end of the member (carry-over factor ½).
7. **Repeat** balancing and carrying over till the carry-over moments become negligible.
8. **Final moments** = algebraic sum of each column. Then find reactions, and draw SFD and BMD.

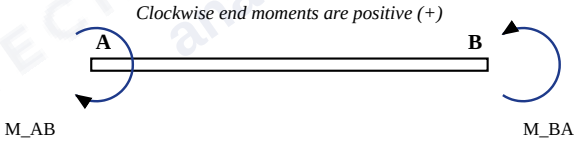


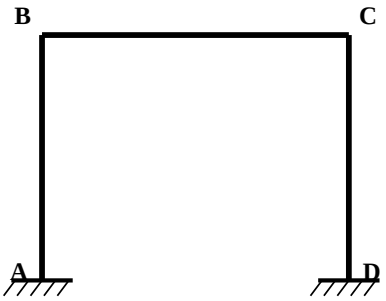
Fig.: Sign convention for member end moments in moment distribution

(Any six steps in sequence, 4 M)

e) Symmetrical and unsymmetrical portal frames 4 M

Symmetrical portal frame	Unsymmetrical portal frame
Frame and loading are symmetrical about the vertical centre line.	Frame or loading (or both) is not symmetrical.
Columns have equal height and equal M.I.; both bases have same end conditions.	Columns of unequal height / M.I. or different end conditions.

No sway (lateral displacement) occurs under vertical loads.	The frame sways sideways; sway moments must be considered.
Analysis is simple (non-sway analysis; half frame can be used).	Analysis is more complex (sway analysis required).



(a) Symmetrical portal frame: equal columns (h, I), same end conditions, symmetrical load

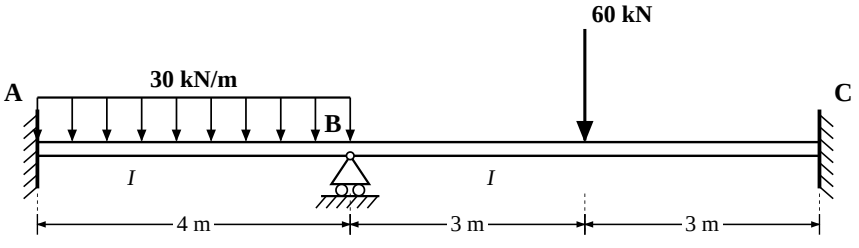
(b) Unsymmetrical portal frame: unequal columns / different supports / unsymmetrical load — frame sways

(Any four points 1 M each (sketches included))

Q.5 Attempt any TWO of the following: (12 M)

a) Three moment theorem — beam fixed at both ends

6 M



Step 1 — Free bending moments and 6Ax̄/L terms (each span treated as simply supported). For UDL: $6A\bar{x}/L = wL^3/4$; for point load W at distance a from the left end: $6A\bar{x}/L = Wab(L + a)/L$ about the left end and $Wab(L + b)/L$ about the right end.

Span	L (m)	Max free BM (kN·m)	Area A	x̄ from left (m)	x̄ from right (m)	$6A\bar{x}_{left}/L$	$6A\bar{x}_{right}/L$
AB	4	$wL^2/8 = 30\times4^2/8 = 60$	160	2	2	480	480
BC	6	$Wab/L = 60\times3\times3/6 = 90$	270	3	3	810	810

Step 2 — Clapeyron's equations (support moments M taken hogging positive):

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = 6A_1\bar{x}_1/L_1 + 6A_2\bar{x}_2/L_2$$

($\bar{x}_1$ measured from the left support of the left span, $\bar{x}_2$ from the right support of the right span.)

Since end A is fixed, an **imaginary span A_0A** of zero length ($L_0 = 0$, no load, $M_{A0} = 0$) is added to the left of A.

Since end C is fixed, an **imaginary span CC_0** of zero length is added to the right of C.

For supports **A_0, A, B**: $8\,M_A + 4\,M_B = 480$ … (1)

For supports **A, B, C**: $4\,M_A + 20\,M_B + 6\,M_C = 1290$ … (2)

For supports **B, C, C_0**: $6\,M_B + 12\,M_C = 810$ … (3)

Solving the simultaneous equations: **M_A = 38.5 kN·m**, **M_B = 43 kN·m**, **M_C = 46 kN·m** (hogging).

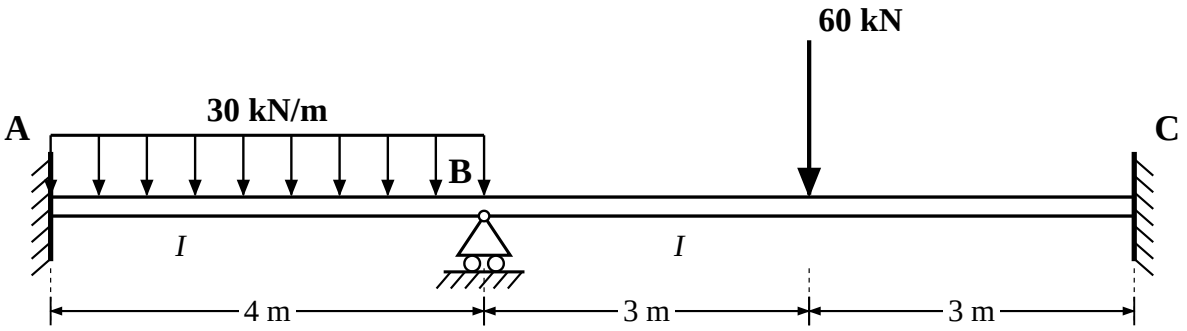
Step 3 — Reactions, SFD and BMD

Support reactions (span-wise: free reaction ± $(M_{left} - M_{right})/L$):

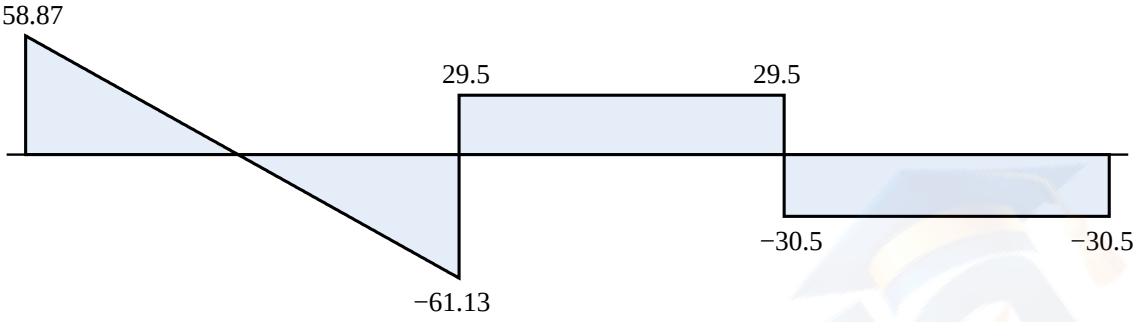
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (4 m)	60 / 60	$(38.5 - 43)/4 = -1.13$	58.87	61.13
BC (6 m)	30 / 30	$(43 - 46)/6 = -0.5$	29.5	30.5

R_A = 58.87 kN; **R_B = 90.63 kN**; **R_C = 30.5 kN** (ΣR = 180 kN = total load ✓)

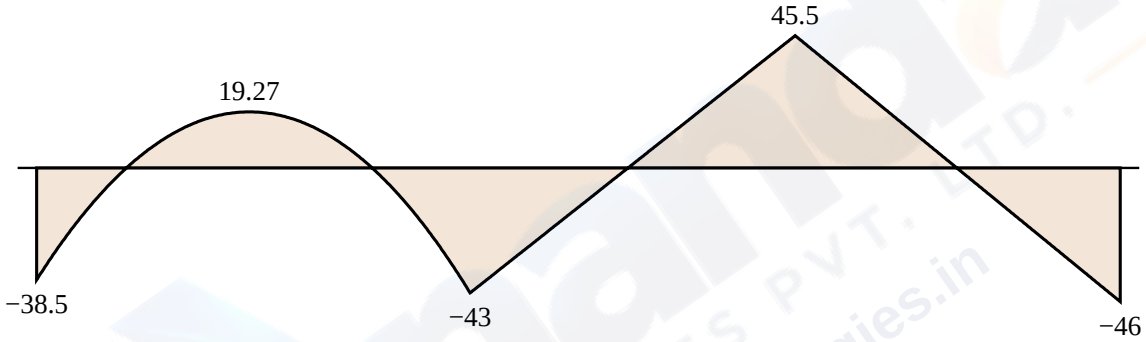
Maximum sagging moments — span AB: **19.27 kN·m** at 1.96 m from A; span BC: **45.5 kN·m** at 3 m from B.



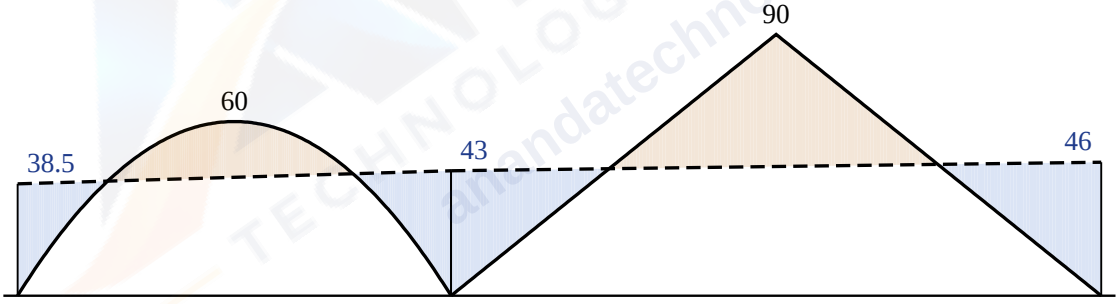
SFD (kN)



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)

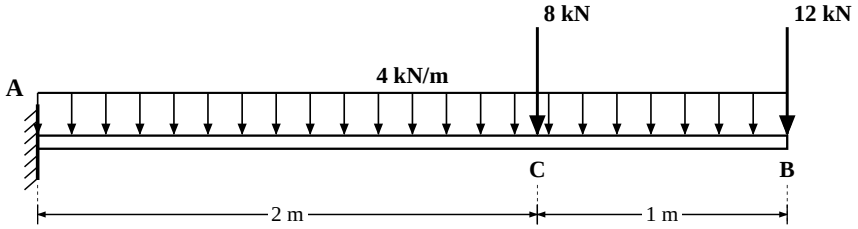


SFD, BMD (net) and BMD by superposition (+ sagging, − hogging)

✔ Check: support moments and reactions agree with an independent stiffness-matrix (and moment distribution) solution.

(Imaginary spans and three equations 2 M, support moments 2 M, SFD 1 M, BMD 1 M)

b) Macaulay's method — cantilever 6 M



Macaulay's method — take the **free end B as origin** and section X–X at distance x from B in the last segment (towards the fixed end A). Bending moment of a cantilever is hogging (negative).

$EI\ d^2y/dx^2 = M_x = -12x - 2x^2 \mid -8[x-1]$

$EI\ dy/dx = -6x^2 - 0.667x^3 \mid -4[x-1]^2 + C_1$ … (1)

$EI\ y = -2x^3 - 0.167x^4 \mid -1.333[x-1]^3 + C_1x + C_2$ … (2)

Boundary conditions at the fixed end A (x = 3 m): $dy/dx = 0 \rightarrow \mathbf{C_1 = 88}$; $y = 0 \rightarrow \mathbf{C_2 = -185.833}$

- At B (x = 0 m from free end): $EI\,\theta = 88 \rightarrow \mathbf{\theta_B = 88/EI = 0.00880\ rad}$; $EI\,y = -185.833 \rightarrow \mathbf{y_B = 185.83/EI = 18.58\ mm}$ (downward)
- At C (x = 1 m from free end): $EI\,\theta = 81.333 \rightarrow \mathbf{\theta_C = 81.33/EI = 0.00813\ rad}$; $EI\,y = -100 \rightarrow \mathbf{y_C = 100/EI = 10.00\ mm}$ (downward)

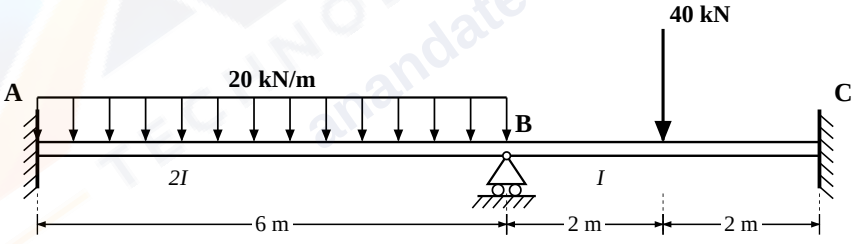
($EI = 200{,}000 \times 5 \times 10^7\ N{\cdot}mm^2 = 10000\ kN{\cdot}m^2$)

For a cantilever, the maximum slope and maximum deflection occur at the **free end**.

✔ Check: slope and deflection agree with an independent stiffness-matrix analysis.

(Macaulay equation 2 M, constants 1 M, slope and deflection at B 2 M, at C 1 M)

c) Moment distribution — beam fixed at both ends 6 M



Step 1 — Fixed end moments (clockwise +, anticlockwise −):

Span	Left end formula	M_{left} (kN·m)	Right end formula	M_{right} (kN·m)
AB	$wL^2/12 = 20\times6^2/12$	**−60**	$wL^2/12$	**+60**
BC	$WL/8 = 40\times4/8$	**−20**	$WL/8$	**+20**

Step 2 — Stiffness and distribution factors (k = I/L for far end fixed or continuous; k = (3/4)·I/L when the far end is simply supported):

Joint	Member	Relative stiffness k	Σk	Distribution factor

A	AB	2I/6 = 0.3333I	0.3333I	0 (fixed end)
B	BA	2I/6 = 0.3333I	0.5833I	0.3333/0.5833 = **0.571**
	BC	I/4 = 0.25I		0.25/0.5833 = **0.429**
C	CB	I/4 = 0.25I	0.25I	0 (fixed end)

Step 3 — Moment distribution table (carry-over factor ½; no carry-over to a simply supported end):

Step	A	B		C
	AB	**BA**	**BC**	**CB**
D.F.	0	0.571	0.429	0
FEM	−60	60	−20	20
Balance		−22.86	−17.14	
Carry over	−11.43			−8.57
Final moments	**−71.43**	**37.14**	**−37.14**	**11.43**

Support moments: M_A = **71.43 kN·m**; M_B = **37.14 kN·m**; M_C = **11.43 kN·m** (hogging; final end moments at a joint are equal and opposite ✓).

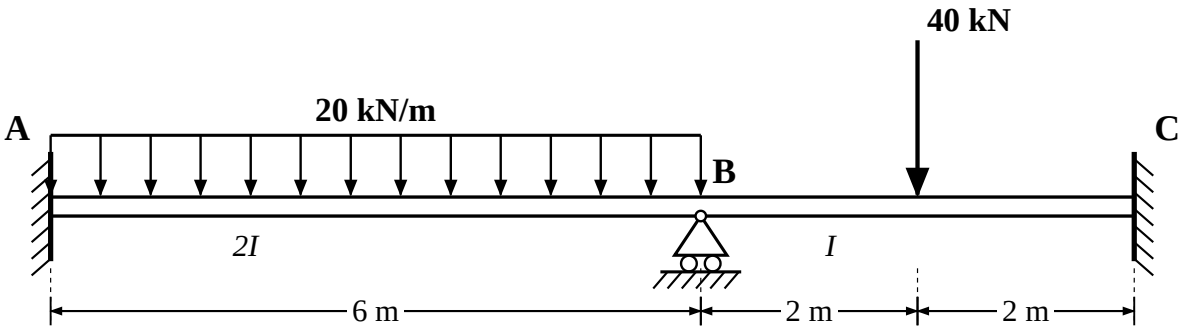
Step 4 — Reactions and diagrams

Support reactions (span-wise: free reaction ± $(M_{left} - M_{right})/L$):

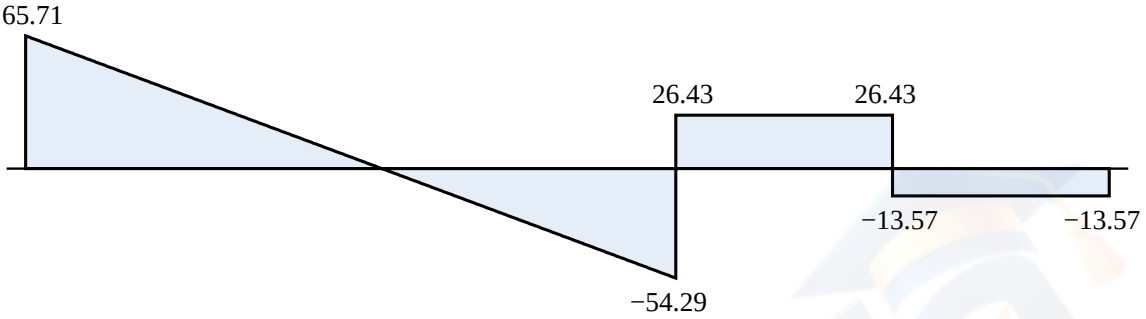
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (6 m)	60 / 60	(71.43 − 37.14)/6 = 5.71	65.71	54.29
BC (4 m)	20 / 20	(37.14 − 11.43)/4 = 6.43	26.43	13.57

R_A = 65.71 kN; **R_B = 80.71 kN**; **R_C = 13.57 kN** (ΣR = 160 kN = total load ✓)

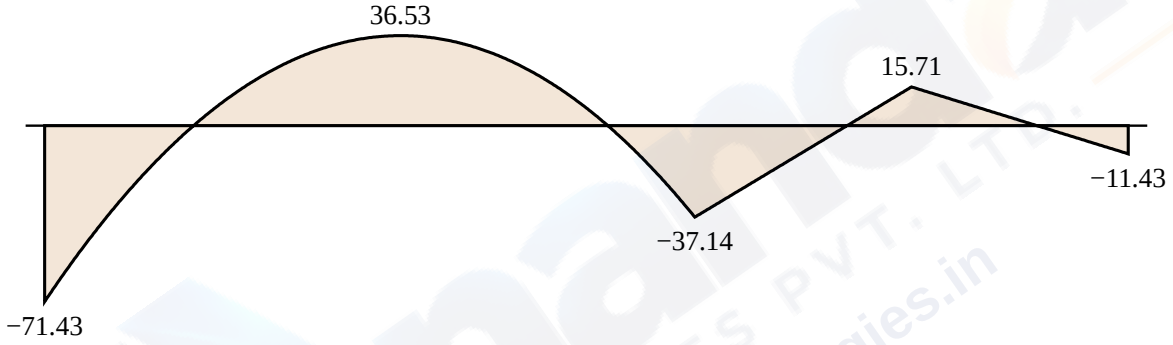
Maximum sagging moments — span AB: **36.53 kN·m** at 3.29 m from A; span BC: **15.71 kN·m** at 2 m from B.



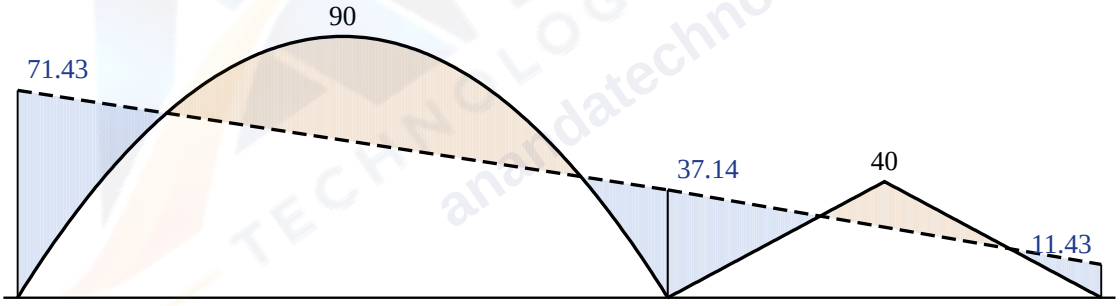
SFD (kN)



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



SFD, BMD (net) and BMD by superposition (+ sagging, − hogging)

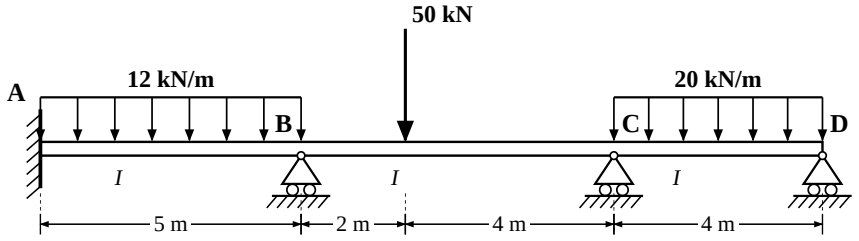
✔ Check: support moments agree with the theorem of three moments and with a stiffness-matrix solve (difference < 0.01 kN·m).

(FEM 1 M, DF 1 M, distribution 2 M, SFD and BMD 2 M)

Q.6 Attempt any TWO of the following: (12 M)

a) Moment distribution — three-span beam

6 M



Step 1 — Fixed end moments (clockwise +, anticlockwise −):

Span	Left end formula	M_{left} (kN·m)	Right end formula	M_{right} (kN·m)
AB	$wL^2/12 = 12\times5^2/12$	**−25**	$wL^2/12$	**+25**
BC	$Wab^2/L^2 = 50\times2\times4^2/6^2$	**−44.44**	$Wa^2b/L^2 = 50\times2^2\times4/6^2$	**+22.22**
CD	$wL^2/12 = 20\times4^2/12$	**−26.67**	$wL^2/12$	**+26.67**

Step 2 — Stiffness and distribution factors (k = I/L for far end fixed or continuous; k = (3/4)·I/L when the far end is simply supported):

Joint	Member	Relative stiffness k	Σk	Distribution factor
A	AB	I/5 = 0.2I	0.2I	0 (fixed end)
B	BA	I/5 = 0.2I	0.3667I	0.2/0.3667 = **0.545**
	BC	I/6 = 0.1667I		0.1667/0.3667 = **0.455**
C	CB	I/6 = 0.1667I	0.3542I	0.1667/0.3542 = **0.471**
	CD	(3/4)·I/4 = 0.1875I		0.1875/0.3542 = **0.529**
D	DC	I/4 = 0.25I	0.25I	1 (simple end)

Step 3 — Moment distribution table (carry-over factor ½; no carry-over to a simply supported end; the simple end is first released and half of the released moment is carried over):

Step	A	B		C		D
	AB	BA	BC	CB	CD	DC
D.F.	0	0.545	0.455	0.471	0.529	1
FEM	−25	25	−44.44	22.22	−26.67	26.67
Release simple end						−26.67
Carry over					−13.33	
Balance		10.61	8.84	8.37	9.41	
Carry over	5.3		4.18	4.42		
Balance		−2.28	−1.9	−2.08	−2.34	
Carry over	−1.14		−1.04	−0.95		
Balance		0.57	0.47	0.45	0.5	
Carry over	0.28		0.22	0.24		

Balance		−0.12	−0.1	−0.11	−0.13	
Carry over	−0.06		−0.06	−0.05		
Balance		0.03	0.03	0.02	0.03	
Carry over	0.02		0.01	0.01		
Final moments	**−20.6**	**33.8**	**−33.79**	**32.54**	**−32.52**	**0**

Support moments: M_A = **20.6 kN·m**; M_B = **33.8 kN·m**; M_C = **32.54 kN·m**; M_D = **0 kN·m** (hogging; final end moments at a joint are equal and opposite ✓).

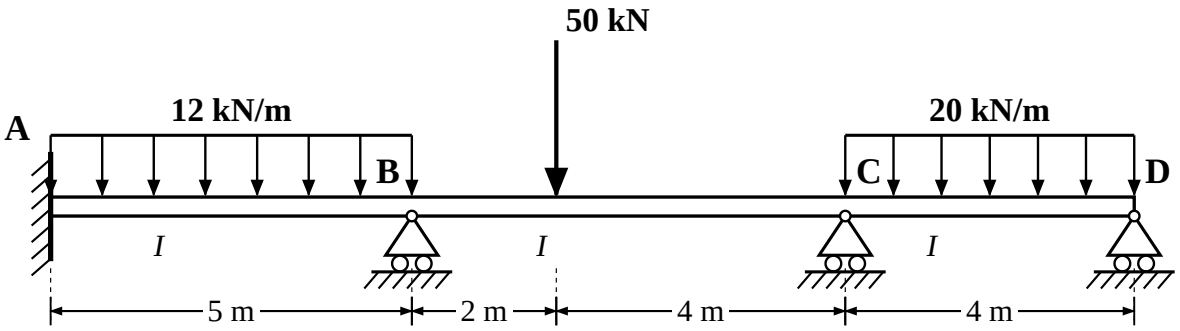
Step 4 — Reactions and diagrams

Support reactions (span-wise: free reaction ± $(M_{left} - M_{right})/L$):

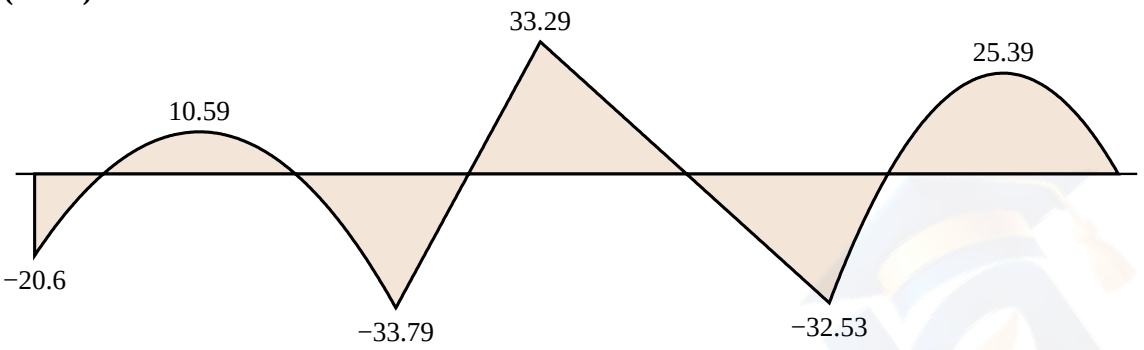
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (5 m)	30 / 30	(20.6 − 33.8)/5 = −2.64	27.36	32.64
BC (6 m)	33.33 / 16.67	(33.8 − 32.54)/6 = 0.21	33.54	16.46
CD (4 m)	40 / 40	(32.54 − 0)/4 = 8.13	48.13	31.87

R_A = 27.36 kN; **R_B = 66.18 kN**; **R_C = 64.59 kN**; **R_D = 31.87 kN** (ΣR = 190 kN = total load ✓)

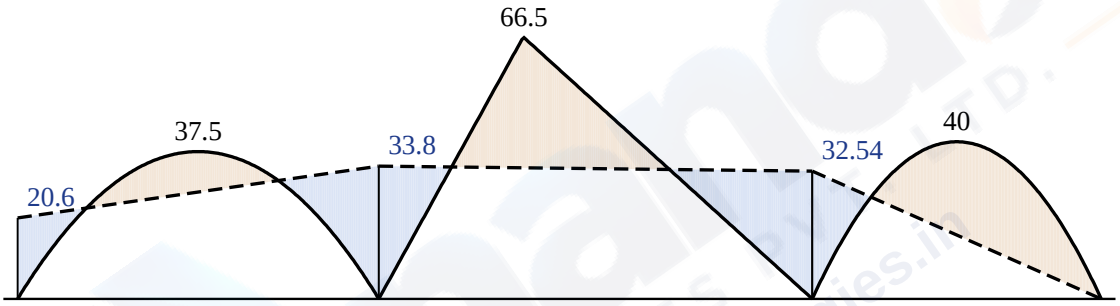
Maximum sagging moments — span AB: **10.59 kN·m** at 2.28 m from A; span BC: **33.28 kN·m** at 2 m from B; span CD: **25.39 kN·m** at 2.41 m from C.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)

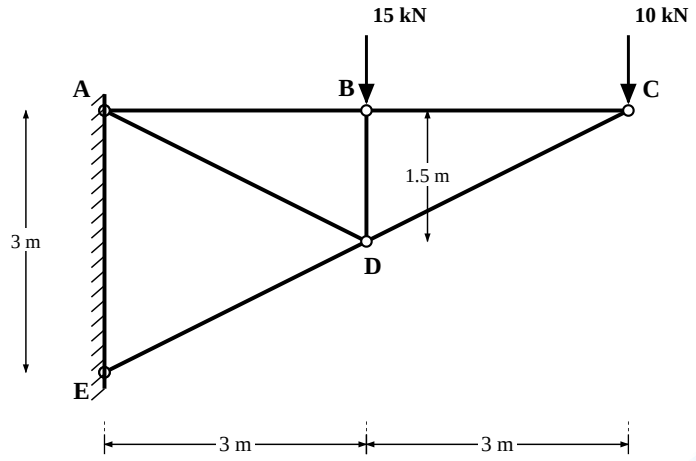


BMD (net) and BMD by superposition (+ sagging, − hogging)

✔ Check: support moments agree with the theorem of three moments and with a stiffness-matrix solve (difference < 0.01 kN·m).

(FEM 1 M, DF 1 M, distribution table 2 M, support moments 1 M, BMD 1 M)

b) Cantilever truss — method of joints 6 M



Check: number of joints j = 5, members m = 6; reaction components r = 4; m + r = 10 = 2j → statically determinate.

For a cantilever truss the member forces can be found starting from the free end C, without first finding the reactions.

All unknown member forces are first assumed **tensile** (acting away from the joint). A negative answer means the member is in **compression**.

Joint C (unknowns: F_{CB}, F_{CD}):

$\Sigma F_x = 0$: $-F_{CB} - F_{CD}\cos 26.57° = 0$

$\Sigma F_y = 0$: $-F_{CD}\sin 26.57° - 10 = 0$

→ F_{CB} = **20 kN (T)**, F_{CD} = **−22.36 kN (C)**

Joint B (unknowns: F_{BA}, F_{BD}):

$\Sigma F_x = 0$: $-F_{BA} + (20) = 0$

$\Sigma F_y = 0$: $-F_{BD} - 15 = 0$

→ F_{BA} = **20 kN (T)**, F_{BD} = **−15 kN (C)**

Joint D (unknowns: F_{DE}, F_{DA}):

$\Sigma F_x = 0$: $(-22.36)\cos 26.57° - F_{DE}\cos 26.57° - F_{DA}\cos 26.57° = 0$

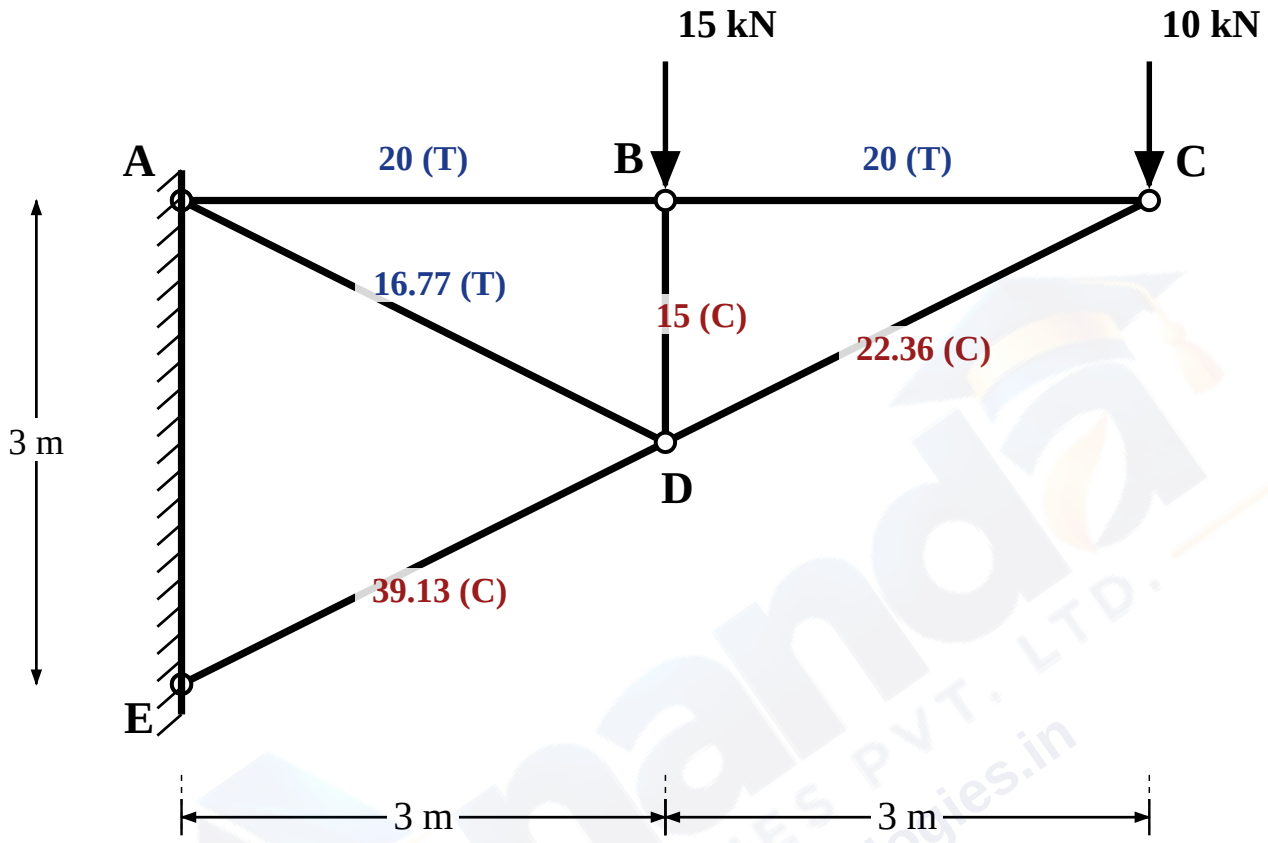
$\Sigma F_y = 0$: $(-22.36)\sin 26.57° - F_{DE}\sin 26.57° + (-15) + F_{DA}\sin 26.57° = 0$

→ F_{DE} = **−39.13 kN (C)**, F_{DA} = **16.77 kN (T)**

Result table

Member	Force (kN)	Nature
AB	20	Tension
BC	20	Tension
CD	22.36	Compression

DE	39.13	Compression
BD	15	Compression
AD	16.77	Tension



Member forces (T = tension, C = compression)

✔ Check: all member forces agree with a full matrix solution of the 2j joint-equilibrium equations.

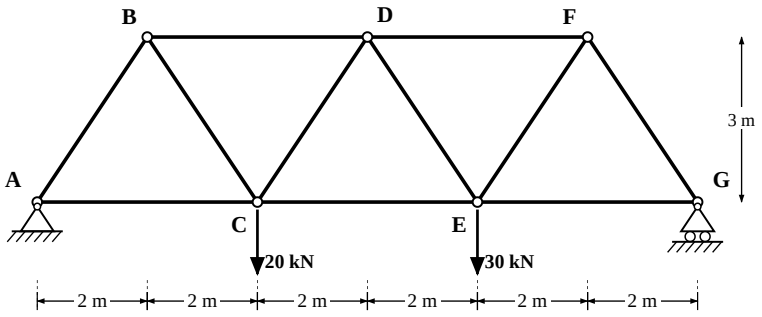
Reactions (from equilibrium of joints A and E, using the member forces found):

Joint A: $H_A = F_{AB} + F_{AD}\cos 26.57° = 20 + 16.77 \times 0.8944 =$ **35 kN (←, towards wall)**; $V_A = F_{AD}\sin 26.57° =$ **7.5 kN (↑)**

Joint E: $H_E =$ **35 kN (→, away from wall)**; $V_E =$ **17.5 kN (↑)**. Check: $V_A + V_E = 25$ kN = 15 + 10 ✓; $H_A + H_E = 0$ ✓; ΣM_E: $H_A \times 3 = 15 \times 3 + 10 \times 6 = 105 \rightarrow H_A = 35$ kN ✓

(Joint equations 3 M, results with nature 2 M, reactions 1 M)

c) Truss — method of sections 6 M



Support reactions: $\Sigma M_A = 0$: $R_G \times 12 = 20 \times 4 + 30 \times 8 \rightarrow$ **R_G = 26.67 kN (↑)**; $\Sigma F_y = 0$: $R_A = 50 - 26.67$ = **23.33 kN (↑)**; $\Sigma F_x = 0$: $H_A = 0$ (all loads vertical).

Draw section line 1–1 cutting members BD, CD, CE and consider the equilibrium of the part containing joints A, B, C. Forces in cut members are assumed tensile (away from the part).

Force in BD: take moments about **C** (anticlockwise +). Perpendicular distance of BD from this point = 3 m.

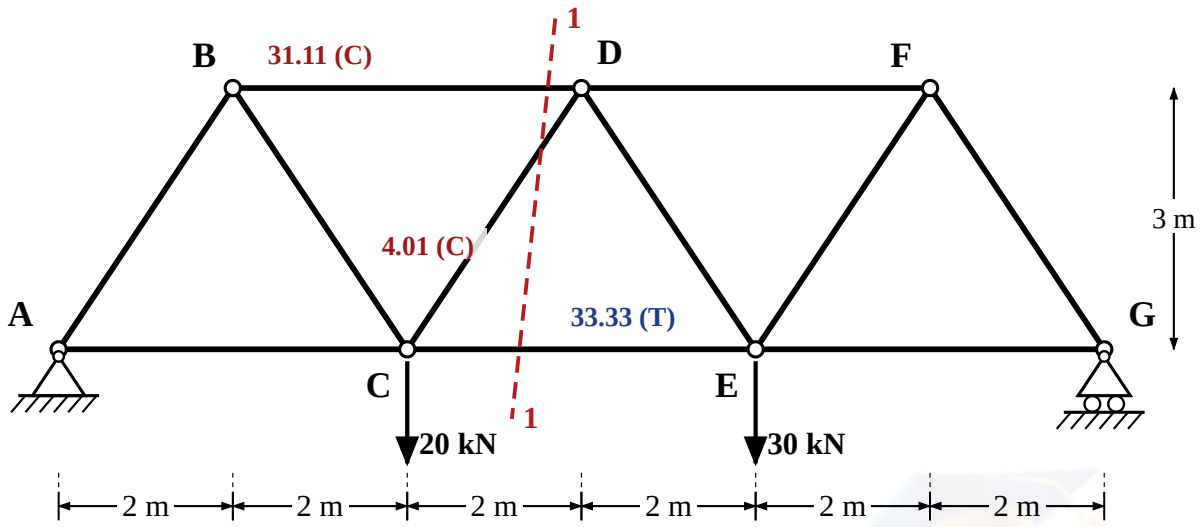
$\Sigma M = 0$: $-23.33 \times 4 - F_{BD} \times 3 = 0 \rightarrow$ **F_{BD} = −31.11 kN (C)**

Force in CD: members BD and CE are horizontal (parallel), so resolve vertically: $\Sigma F_y = 0$: $23.33 - 20 + F_{CD} \sin 56.31° = 0 \rightarrow$ **F_{CD} = −4.01 kN (C)**

Force in CE: take moments about **D** (anticlockwise +). Perpendicular distance of CE from this point = 3 m.

$\Sigma M = 0$: $-23.33 \times 6 + 20 \times 2 + F_{CE} \times 3 = 0 \rightarrow$ **F_{CE} = 33.33 kN (T)**

Member	Force (kN)	Nature
BD	31.11	Compression
CD	4.01	Compression
CE	33.33	Tension



Section 1–1 and forces in the cut members

✔ Check: values agree with the full matrix (method of joints) solution of the truss.

(Reactions 1 M, section 1 M, each force with nature 4 M)

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