Circuits and Networks (313325)

Model Question Paper — Set C · Model Answers

Q.1

a) Linear and bilateral network. 2 M

i) Linear network: A network whose elements (R, L, C) have constant parameters, i.e. they do not change with voltage or current; it obeys Ohm's law and the principle of superposition (V-I characteristic is a straight line). Example: resistor network.

ii) Bilateral network: A network whose elements offer the same property (same impedance) for current flowing in either direction. Example: R, L, C. (A diode is unilateral.)

(Each 1 M)

b) Reciprocity theorem. 2 M

In any linear, bilateral, passive network containing a single source, the ratio of the voltage V in one branch to the current I produced in another branch remains the same when the positions of the source and the response (ammeter) are interchanged. i.e. $V_1/I_2 = V_2/I_1$.

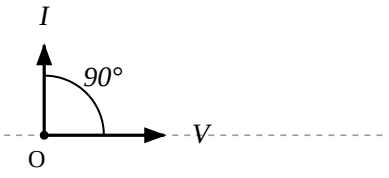
c) Units. 2 M

Quantity	Unit
Impedance (Z)	ohm (Ω)
Reactance (X)	ohm (Ω)
Admittance (Y)	siemens (S) or mho (℧)
Susceptance (B)	siemens (S) or mho (℧)

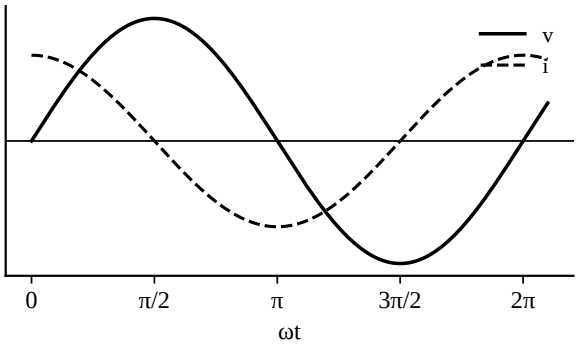
(½ M each)

d) Purely capacitive circuit: phasor diagram and waveforms. 2 M

In a purely capacitive circuit, current **leads** the voltage by 90° ($i = I_m \sin(\omega t + 90°)$).



Phasor diagram

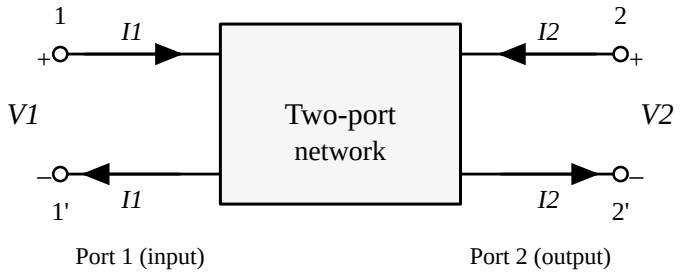


Waveforms (i leads v by 90°)

(Phasor 1 M, waveforms 1 M)

e) Two-port network. 2 M

A network having two pairs of terminals — one pair (port 1) as input and the other pair (port 2) as output — is called a two-port network. The current entering one terminal of a port leaves from the other terminal of the same port. Examples: transistor amplifier, transmission line, filter, attenuator.



Two-port network

f) Q-factor of coil and bandwidth. 2 M

i) Q-factor of a coil: It is the ratio of the inductive reactance of the coil to its resistance at the operating frequency, Q = ωL/R. It indicates the quality (low loss) of the coil.

ii) Bandwidth: The range of frequencies between the two half-power points f_1 and f_2 (where current/voltage response falls to 0.707 of its maximum). BW = $f_2 - f_1 = f_r/Q$.

g) Superposition theorem. 2 M

In any linear, bilateral network containing two or more independent sources, the current through (or voltage across) any element is the algebraic sum of the currents (or voltages) produced by each source acting alone, while all other independent sources are replaced by their internal resistances (ideal voltage sources short-circuited, ideal current sources open-circuited).

Q.2

a) Steps of nodal analysis with example (Fig. No. 1). 4 M

1. Identify all the nodes; select one node (usually the one with most branches) as reference (ground) node.
2. Assign node voltages V1, V2, … to the remaining (non-reference) nodes with respect to the reference.
3. Apply KCL at each non-reference node, expressing branch currents in terms of node voltages: I = $(V_{node} - V_{other})/R$.
4. Solve the simultaneous equations (substitution / Cramer's rule) to get node voltages.
5. Find the required branch currents from node voltages.

Example (Fig. No. 1): one unknown node V (bottom = reference). KCL, sum of currents leaving the node = 0:

$(V - 10)/2 + V/4 - 2 = 0$

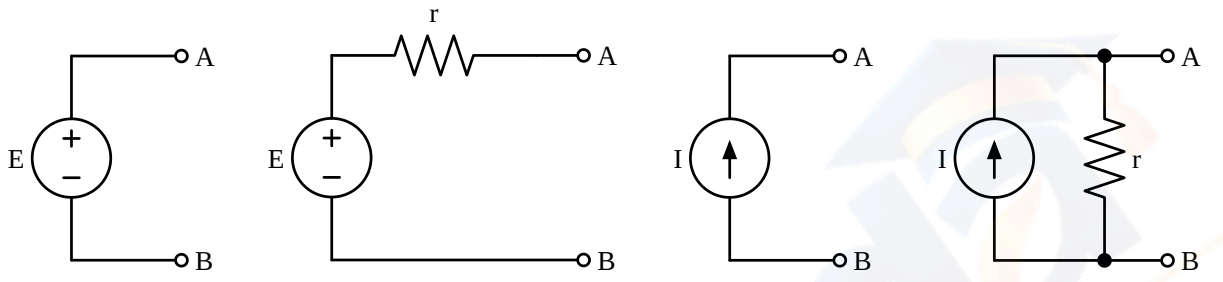
$\Rightarrow 3V = 28 \quad \Rightarrow \quad \mathbf{V = 9.3333\ V}$

Current through 4 Ω = V/4 = **2.3333 A**; current from 10 V source = (10 − V)/2 = 0.3333 A

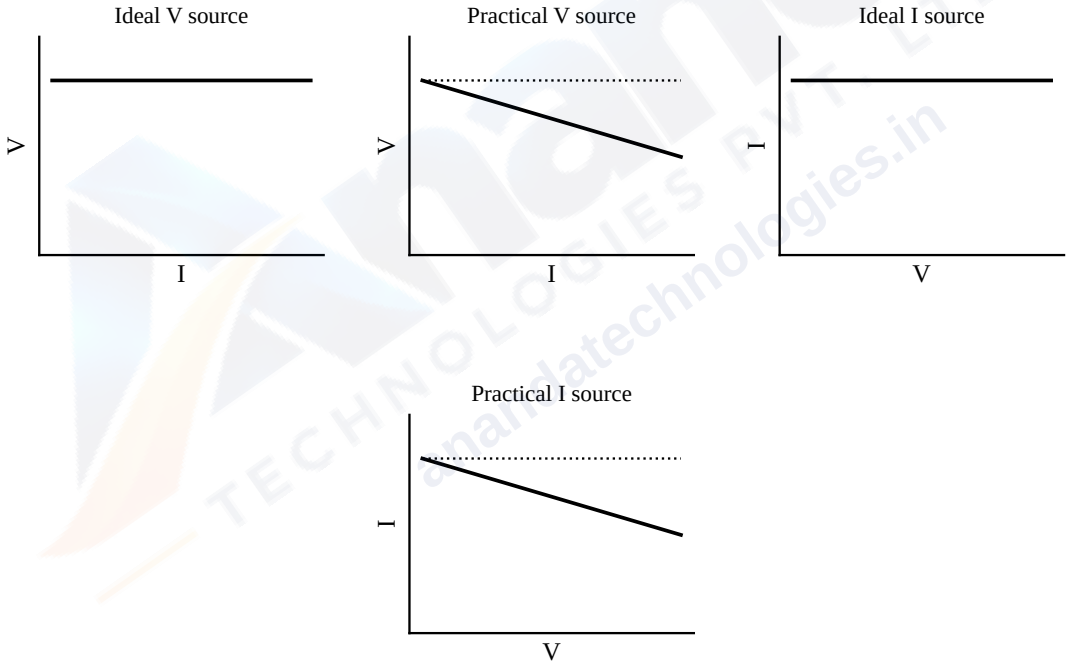
(Steps 2 M, example 2 M)

b) Ideal and practical voltage and current sources. 4 M

- **Ideal voltage source:** gives a constant terminal voltage irrespective of the load current; internal resistance = 0.
- **Practical voltage source:** has a small internal resistance r in **series**; terminal voltage V = E − Ir falls as load current increases.
- **Ideal current source:** gives a constant current irrespective of the terminal voltage (load); internal resistance = ∞.
- **Practical current source:** has a large internal resistance r in **parallel**; load current $I_L = I - V/r$ falls as terminal voltage increases.



(a) Ideal voltage source *(b) Practical voltage source* *(c) Ideal current source* *(d) Practical current source*

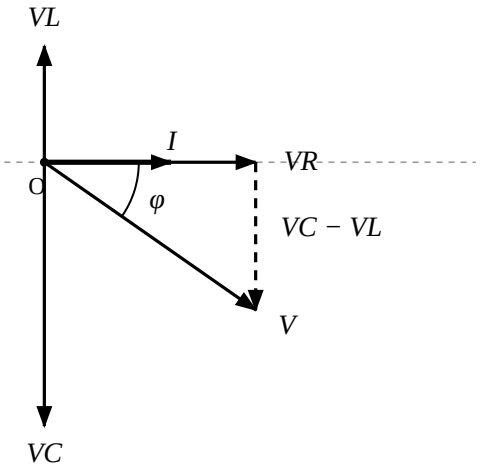


V-I characteristics (dotted line = ideal)

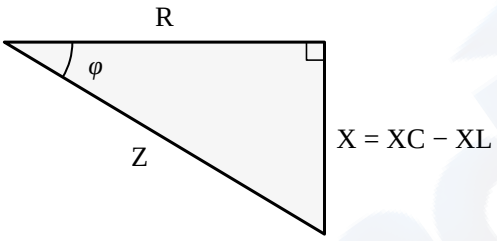
(Explanation 2 M, symbols and characteristics 2 M)

c) Series RLC ($X_L < X_C$): phasor diagram and triangles. 4 M

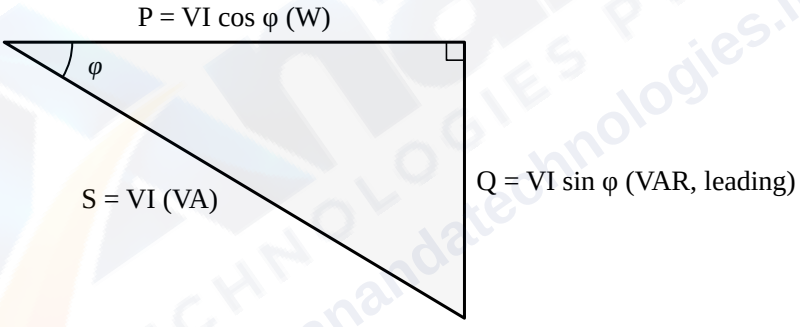
When $X_L < X_C$, $V_C > V_L$; the circuit is **capacitive** and the current I **leads** the supply voltage V by φ. Current I is the reference phasor.



Phasor diagram (XL < XC, I leads V by φ)



Impedance triangle



Power triangle

$Z = \sqrt{(R^2 + (X_C - X_L)^2)}$; cos φ = R/Z (leading).

(Phasor 2 M, triangles 2 M)

d) ABCD (transmission) parameters. 4 M

The input quantities (V_1, I_1) are expressed in terms of output quantities (V_2, $-I_2$):

$V_1 = A\,V_2 - B\,I_2$

$I_1 = C\,V_2 - D\,I_2$

Parameter	Definition	Name / unit
A	V_1/V_2 with $I_2 = 0$ (port 2 open)	Open-circuit reverse voltage ratio (no unit)

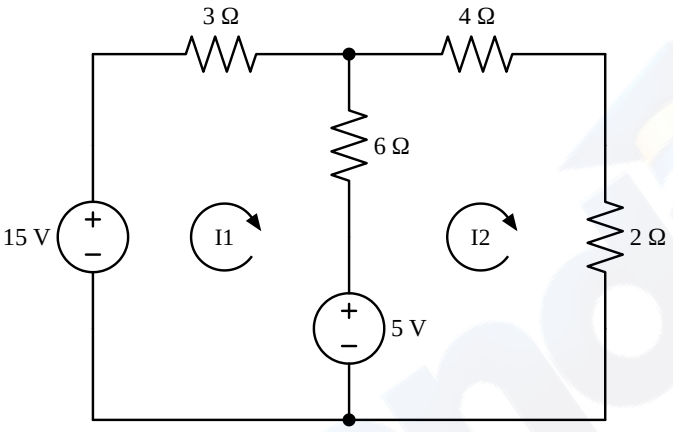
B	$V_1/(-I_2)$ with $V_2 = 0$ (port 2 short)	Short-circuit transfer impedance (Ω)
C	I_1/V_2 with $I_2 = 0$	Open-circuit transfer admittance (S)
D	$I_1/(-I_2)$ with $V_2 = 0$	Short-circuit reverse current ratio (no unit)

Reciprocity: AD − BC = 1 **Symmetry:** A = D. (Used for cascaded networks and transmission lines.)

(Equations 1 M, definitions 2 M, conditions 1 M)

Q.3

a) Mesh analysis — current through 6 Ω (Fig. No. 2). 4 M



Clockwise mesh currents I1 and I2. The middle branch (6 Ω with 5 V source, + at top) is common to both meshes. In mesh 1 the current I1 flows down through the 5 V source (+ to −), so it is a drop of 5 V; in mesh 2, I2 flows up through it (− to +), a rise of 5 V.

Mesh 1: 3I1 + 6(I1 − I2) = 15 − 5 ⟹ 9I1 − 6I2 = 10 ……(1)

Mesh 2: 6(I2 − I1) + 4I2 + 2I2 = 5 ⟹ −6I1 + 12I2 = 5 ……(2)

Writing in matrix form:

$$\begin{vmatrix} 9 & -6 \\ -6 & 12 \end{vmatrix} \times \begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} 10 \\ 5 \end{vmatrix}$$

Δ = | 9 −6 ; −6 12 | = **72**

Δ1 (replace column 1 by constants) = | 10 −6 ; 5 12 | = **150** ⟹ I1 = Δ1/Δ = 150/72 = **2.0833 A**

Δ2 (replace column 2 by constants) = | 9 10 ; −6 5 | = **105** ⟹ I2 = Δ2/Δ = 105/72 = **1.4583 A**

Current through 6 Ω = I1 − I2 = 2.0833 − 1.4583 = **0.625 A** (downward)

(Equations 2 M, solution 2 M)

b) Maximum power transfer (Fig. No. 3). 4 M

R_{th}: Remove R_L; open-circuit the 6 A current source. Seen from load terminals: 5 Ω in series with 10 Ω:

R_{th} = 5 + 10 = **15 Ω**

V_{th}: With R_L removed no current flows in 5 Ω, so the whole 6 A flows through 10 Ω:

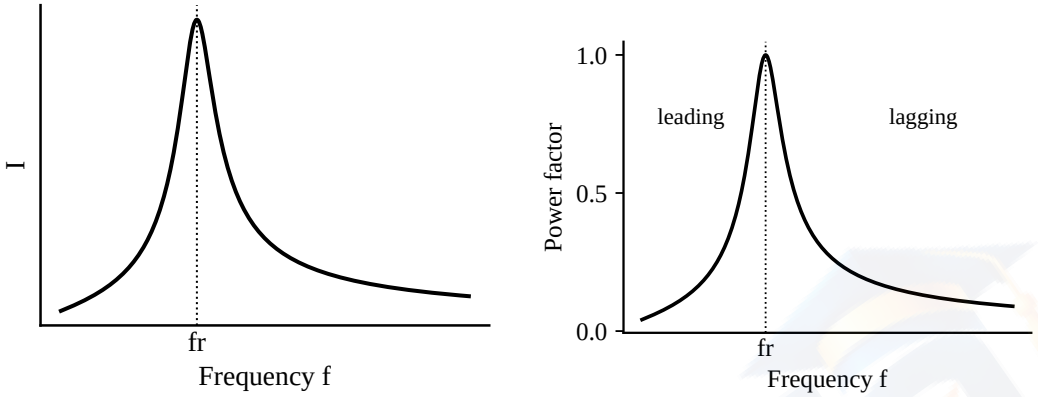
$V_{th} = 6 \times 10 =$ **60 V**

For maximum power: $\mathbf{R_L = R_{th} = 15\ \Omega}$

$P_{max} = V_{th}^2/(4R_{th}) = 60^2/(4 \times 15) =$ **60 W**

(Rth 1 M, Vth 1 M, RL 1 M, Pmax 1 M)

c) Variation of current and power factor with frequency in series RLC. 4 M



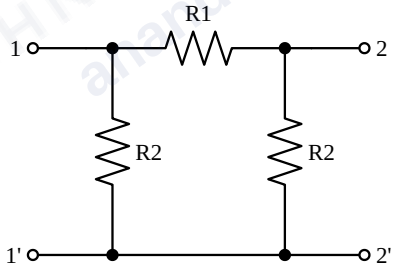
(a) Current vs frequency *(b) Power factor vs frequency*

(Curves computed for R = 20 Ω, L = 0.1 H, C = 10 μF)

- **Current:** I = V/Z. At low frequency X_C is very large, so Z is large and current is small. As f increases, Z decreases and I increases; at f = f_r, Z = R (minimum) and current is **maximum, I = V/R**. Above f_r, X_L increases, Z increases and current again decreases.
- **Power factor:** cos φ = R/Z. Below f_r the circuit is capacitive and pf is **leading**; it increases towards unity as f approaches f_r. At f_r, **pf = 1** (Z = R). Above f_r the circuit is inductive and pf is **lagging**, decreasing as f increases.

(Curves 2 M, explanation 2 M)

d) Symmetrical π attenuator: circuit, features, applications. 4 M



Symmetrical π attenuator (R0 = characteristic resistance)

Design equations: $R1 = R0(N^2 - 1)/(2N)$, $R2 = R0(N + 1)/(N - 1)$, where $N = V_{in}/V_{out}$ (attenuation in dB = 20 log N).

Features:

- Consists of one series arm and two equal shunt arms; made of pure resistors, so attenuation is independent of frequency.

- Input and output impedances are equal to the characteristic resistance R0 (matched on both sides).
- Fixed attenuation; it is the delta (Δ) equivalent of T attenuator; balanced form is the O-attenuator.

Applications: reducing signal level in communication / audio lines, impedance matching between stages, in measuring instruments and signal generators (step attenuators), protecting inputs from overload.

(Circuit 1 M, features 2 M, applications 1 M)

Q.4

a) Equivalent resistance between A and B (Fig. No. 4). 4 M

Let the top node be C and bottom node be D. Check the bridge balance condition:

$R_{AC}/R_{CB} = 4/8 = 0.5$ and $R_{AD}/R_{DB} = 6/12 = 0.5$

Since the ratios are equal, the bridge is **balanced**: $V_C = V_D$, so no current flows through the 10 Ω resistor and it can be removed.

Upper path A–C–B = 4 + 8 = 12 Ω; lower path A–D–B = 6 + 12 = 18 Ω

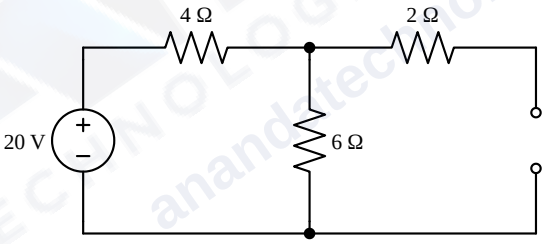
$R_{AB} = 12 \parallel 18 = (12 \times 18)/(12 + 18) =$ **7.2 Ω**

(Same result is obtained by star-delta transformation; the 10 Ω carries zero current.)

(Balance check 2 M, calculation 2 M)

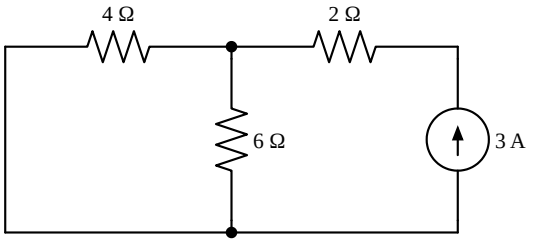
b) Superposition — current through 6 Ω (Fig. No. 5). 4 M

Case 1: 20 V source alone (3 A current source open-circuited). The 2 Ω branch is then open, so 4 Ω and 6 Ω are in series:



I' = 20/(4 + 6) = **2 A** ↓

Case 2: 3 A source alone (20 V source short-circuited). 3 A flows through 2 Ω and divides between 6 Ω and 4 Ω (4 Ω is now connected to ground):



I'' = 3 × 4/(4 + 6) = **1.2 A** ↓

Total current through 6 Ω: I = I' + I'' = 2 + 1.2 = **3.2 A** (downward)

(Each case 1.5 M, total 1 M)

c) Y-parameters of π-network (Fig. No. 6). 4 M

Admittances: $Y_a = 1/2 = 0.5$ S (shunt at port 1), $Y_c = 1/4 = 0.25$ S (series), $Y_b = 1/5 = 0.2$ S (shunt at port 2).

(i) Port 2 short-circuited ($V_2 = 0$): 2 Ω and 4 Ω are in parallel across port 1; 5 Ω is shorted.

$I_1 = V_1(1/2 + 1/4) \Longrightarrow Y_{11} = I_1/V_1 =$ **0.75 S**

Current through 4 Ω into the short $= V_1/4$, so $I_2 = -V_1/4 \Longrightarrow Y_{21} =$ **−0.25 S**

(ii) Port 1 short-circuited ($V_1 = 0$):

$I_2 = V_2(1/5 + 1/4) \Longrightarrow Y_{22} =$ **0.45 S**; $I_1 = -V_2/4 \Longrightarrow Y_{12} =$ **−0.25 S**

$$[Y] = \begin{vmatrix} 0.75 & -0.25 \\ -0.25 & 0.45 \end{vmatrix} \text{ S}$$

$Y_{12} = Y_{21}$ → reciprocal; $Y_{11} \neq Y_{22}$ → not symmetrical.

(Each parameter 1 M)

d) Coil: 5 A from 230 V, 50 Hz, 600 W. 4 M

i) $R = P/I^2 = 600/5^2 =$ **24 Ω**

$Z = V/I = 230/5 = 46$ Ω

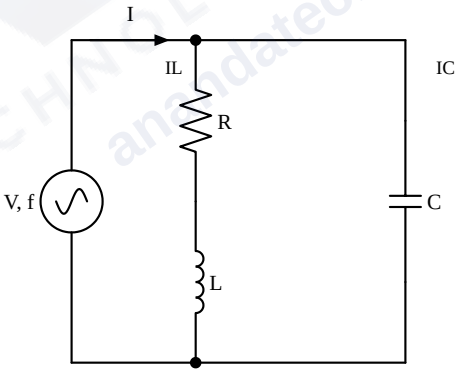
ii) $X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(46^2 - 24^2)} =$ **39.243 Ω**

iii) $L = X_L/(2\pi f) = 39.243/(2\pi \times 50) =$ **0.1249 H** ≈ 124.9 mH

iv) pf $= R/Z = 24/46 =$ **0.5217 lagging** [check: $VI \cos\varphi = 600$ W]

(Each 1 M)

e) Resonant frequency of parallel resonant circuit; dynamic impedance. 4 M



Coil branch current $I_L = V/Z_L$, $Z_L = \sqrt{(R^2 + X_L^2)}$, lagging V by φ_L; capacitor current $I_C = V/X_C$, leading V by 90°.

At resonance the supply current is in phase with V, so the reactive (quadrature) components cancel:

$I_C = I_L \sin\varphi_L \Longrightarrow V/X_C = (V/Z_L)(X_L/Z_L) \Longrightarrow X_L X_C = Z_L^2$

$\omega L \times 1/(\omega C) = R^2 + \omega^2 L^2 \Longrightarrow L/C = R^2 + \omega_r^2 L^2 \Longrightarrow \omega_r^2 = 1/(LC) - R^2/L^2$

$f_r = (1/2\pi)\sqrt{(1/(LC) - R^2/L^2)}$ (if R is negligible, $f_r = 1/(2\pi\sqrt{(LC)})$)

Dynamic impedance: supply current $I = I_L \cos \varphi_L = V\,R/Z_L^2 = V\,R\,C/L$, so

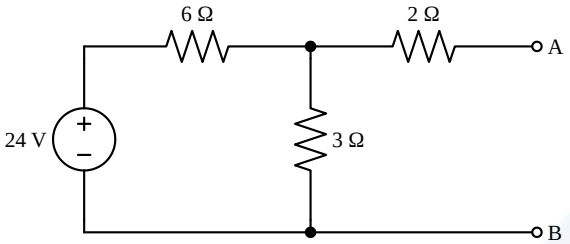
$Z_D = V/I = L/(CR)$ Ω (purely resistive, maximum)

(Derivation 3 M, ZD 1 M)

Q.5

a) Thevenin equivalent and load current for R_L = 2, 4, 6 Ω (Fig. No. 7). 6 M

Step 1 — V_{th}: Remove R_L. No current in 2 Ω, so V_{th} = voltage across 3 Ω:

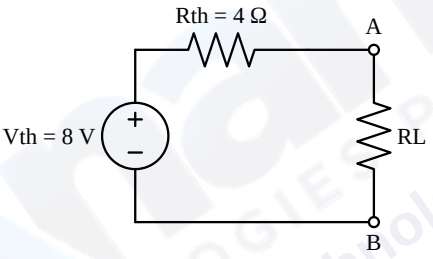


$I = 24/(6 + 3) = 2.6667$ A; $V_{th} = 2.6667 \times 3 =$ **8 V**

Step 2 — R_{th}: Short the 24 V source:

$R_{th} = (6 \parallel 3) + 2 = 2 + 2 =$ **4 Ω**

Step 3 — Thevenin equivalent circuit:



Thevenin's equivalent circuit

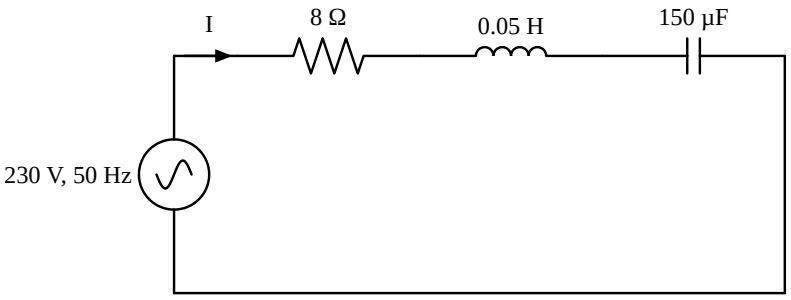
$I_L = V_{th}/(R_{th} + R_L)$:

R_L	$I_L = V_{th}/(R_{th} + R_L)$	I_L (A)
2 Ω	8/(4 + 2)	1.3333
4 Ω	8/(4 + 4)	1
6 Ω	8/(4 + 6)	0.8

(The advantage of Thevenin's theorem: the network is solved only once, and the current for any load is found quickly.)

(Vth 2 M, Rth 1.5 M, equivalent circuit 1 M, three currents 1.5 M)

b) Series R = 8 Ω, L = 0.05 H, C = 150 μF on 230 V, 50 Hz. 6 M



$X_L = 2\pi \times 50 \times 0.05 = 15.708\ \Omega$; $X_C = 1/(2\pi \times 50 \times 150 \times 10^{-6}) = 21.221\ \Omega$; $X_C > X_L \rightarrow$ capacitive

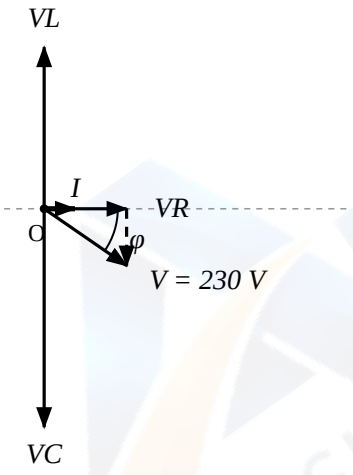
i) $Z = \sqrt{(8^2 + (15.708 - 21.221)^2)} = \sqrt{(8^2 + 5.513^2)} =$ **9.715 Ω**

ii) $I = 230/9.715 =$ **23.674 A**

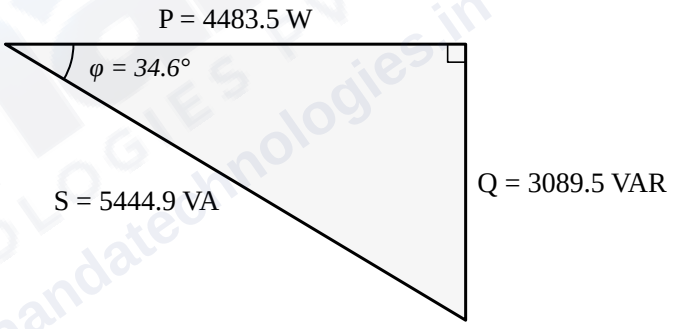
iii) $pf = R/Z = 8/9.715 =$ **0.8234 leading** ($\varphi = 34.57°$)

iv) $P = VI \cos\varphi =$ **4483.53 W**; $Q = VI \sin\varphi = 230 \times 23.674 \times 0.5674 =$ **3089.55 VAR** (leading); $S = VI =$ **5444.94 VA**

$V_R = IR = 189.39$ V, $V_L = IX_L = 371.86$ V, $V_C = IX_C = 502.37$ V.



Phasor diagram (I leads V by φ)



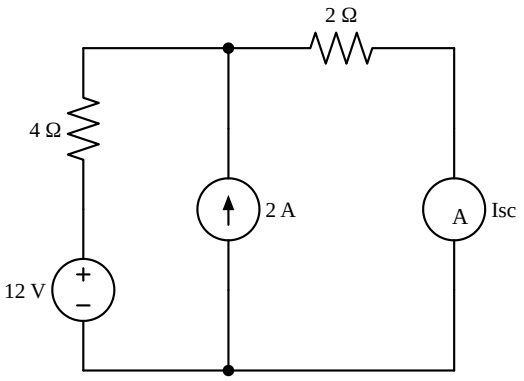
Power triangle (leading)

(Z, I, pf 3 M; powers 1.5 M; phasor and power triangle 1.5 M)

c) Norton's equivalent across A-B (Fig. No. 8); current in 8 Ω. 6 M

Step 1 — I_N (short-circuit current): Short A-B. Convert the 12 V, 4 Ω source into a current source: 12/4 = 3 A in parallel with 4 Ω. It is in parallel with the 2 A source (same direction): Total current source = 3 + 2 = 5 A in parallel with 4 Ω

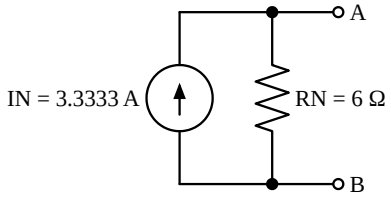
This current divides between 4 Ω and the (2 Ω + short) path: $I_N = 5 \times 4/(4 + 2) =$ **3.3333 A**



Step 2 — R_N: Short the 12 V source, open the 2 A source; seen from A-B: 2 Ω in series with 4 Ω:

$R_N = 2 + 4 =$ **6 Ω**

Step 3 — Norton equivalent:



Norton's equivalent circuit across A-B

With $R_L = 8$ Ω: $I_L = I_N \times R_N/(R_N + R_L) = 3.3333 \times 6/(6 + 8) =$ **1.4286 A**

(IN 2.5 M, RN 1.5 M, circuit 1 M, IL 1 M)

Q.6

a) Nodal analysis — three nodes (Fig. No. 9). 6 M

Bottom node = reference. Unknown node voltages V1, V2, V3. KCL at each node (sum of currents leaving = 0); 5 A enters node 1 and 2 A leaves node 3:

Node 1: $-5 + V1/4 + (V1 - V2)/2 = 0 \implies 3V1 - 2V2 = 20$ ……(1)

Node 2: $V2/6 + (V2 - V1)/2 + (V2 - V3)/3 = 0 \implies -3V1 + 6V2 - 2V3 = 0$ ……(2)

Node 3: $V3/8 + 2 + (V3 - V2)/3 = 0 \implies -8V2 + 11V3 = -48$ ……(3)

(Each equation multiplied by the LCM of its denominators.)

Writing in matrix form:

$$\begin{vmatrix} 3 & -2 & 0 \\ -3 & 6 & -2 \\ 0 & -8 & 11 \end{vmatrix} \times \begin{vmatrix} V1 \\ V2 \\ V3 \end{vmatrix} = \begin{vmatrix} 20 \\ 0 \\ -48 \end{vmatrix}$$

$\Delta = |\ 3\ -2\ 0\ ;\ -3\ 6\ -2\ ;\ 0\ -8\ 11\ | =$ **84**

$\Delta 1$ (replace column 1 by constants) $= |\ 20\ -2\ 0\ ;\ 0\ 6\ -2\ ;\ -48\ -8\ 11\ | =$ **808** $\implies V1 = \Delta 1/\Delta = 808/84 =$ **9.619 V**

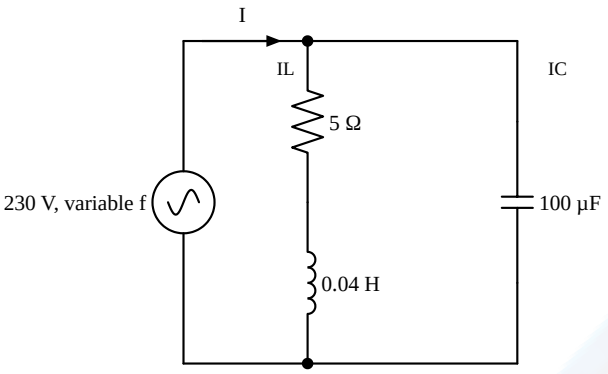
$\Delta 2$ (replace column 2 by constants) $= |\ 3\ 20\ 0\ ;\ -3\ 0\ -2\ ;\ 0\ -48\ 11\ | =$ **372** $\implies V2 = \Delta 2/\Delta = 372/84 =$ **4.4286 V**

$\Delta 3$ (replace column 3 by constants) = | 3 −2 20 ; −3 6 0 ; 0 −8 −48 | = **−96** $\Rightarrow$ V3 = $\Delta 3/\Delta$ = −96/84 = **−1.1429 V**

Power in 6 Ω = $V2^2/6 = (4.4286)^2/6$ = **3.2687 W**

(Three equations 3 M, solution 2 M, power 1 M)

b) Parallel resonance: coil 5 Ω, 0.04 H with 100 µF, 230 V. 6 M



i) $f_r = (1/2\pi)\sqrt{(1/(LC) - R^2/L^2)} = (1/2\pi)\sqrt{(1/(0.04 \times 100 \times 10^{-6}) - 5^2/0.04^2)} = (1/2\pi)\sqrt{(250000 - 15625)} = (1/2\pi) \times 484.123$ = **77.051 Hz**

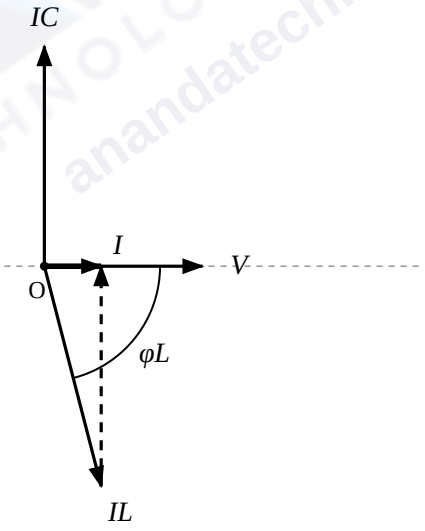
ii) $Z_D = L/(CR) = 0.04/(100 \times 10^{-6} \times 5)$ = **80 Ω**

iii) $I = V/Z_D = 230/80$ = **2.875 A** (in phase with V)

iv) $X_L = \omega_r L = 19.365\ \Omega$, $Z_{coil} = \sqrt{(5^2 + 19.365^2)} = 20\ \Omega \Rightarrow I_L = 230/20$ = **11.5 A**, lagging V by 75.52°

$X_C = 1/(\omega_r C) = 20.656\ \Omega \Rightarrow I_C = 230/20.656$ = **11.1348 A**, leading V by 90°

Check: $I_L \cos \varphi_L = 2.875$ A = supply current; $I_L \sin \varphi_L = 11.1348$ A = I_C ✓



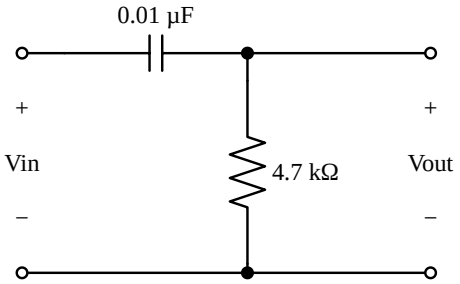
Phasor diagram at resonance (I in phase with V)

(fr 2 M, ZD 1 M, I 1 M, branch currents and phasor 2 M)

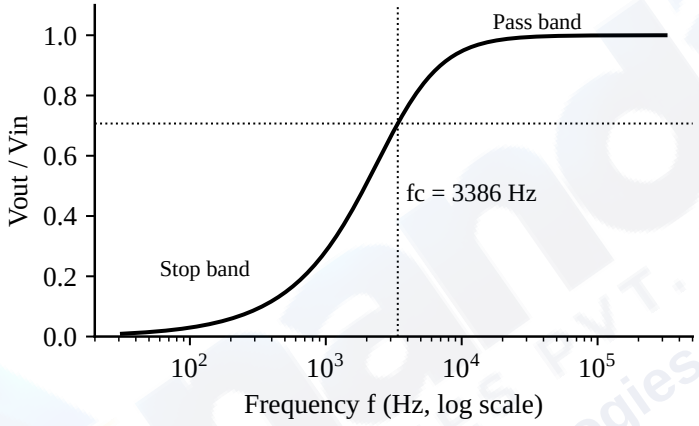
c) RC high pass filter. 6 M

A **high pass filter** passes all frequencies above the cut-off frequency f_c and attenuates frequencies below f_c.

In the RC HPF, C is in the series arm and the output is taken across R. At low frequency X_C is very large, so most of the input drops across C and output is small; at high frequency X_C is small, so nearly all the input appears across R.



RC high pass filter



Frequency response of RC HPF (computed)

$V_{out}/V_{in} = R/\sqrt{(R^2 + X_C^2)}$; at f_c: $X_C = R \Longrightarrow$ **$f_c = 1/(2\pi RC)$**, $V_{out} = 0.707\ V_{in}$

$f_c = 1/(2\pi \times 4.7 \times 10^3 \times 0.01 \times 10^{-6}) =$ **3386.3 Hz ≈ 3.39 kHz**

Applications: i) Removing dc and low-frequency hum/noise from signals. ii) Treble (tweeter) circuits in audio systems. iii) RC coupling between amplifier stages.

(Definition 1 M, circuit 1 M, response 1 M, calculation 2 M, applications 1 M)

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