Circuits and Networks (313325)

Model Question Paper — Set A · Model Answers

Q.1

a) Node and mesh. 2 M

i) Node: A node is a junction point in a network where two or more circuit elements (branches) are connected together.

ii) Mesh: A mesh is a closed loop in a network which does not contain any other loop inside it (i.e. the smallest closed path).

(Each definition 1 M)

b) KCL and KVL. 2 M

KCL: The algebraic sum of currents meeting at a node (junction) is zero, i.e. sum of incoming currents = sum of outgoing currents. $\Sigma I = 0$.

KVL: In any closed loop (mesh) of a network, the algebraic sum of all the emfs and voltage drops (IR products) is zero. $\Sigma V = 0$, i.e. $\Sigma E = \Sigma IR$.

(Each law 1 M)

c) Norton's theorem. 2 M

Any linear, bilateral, active two-terminal network can be replaced by an equivalent circuit consisting of a **current source I_N** in **parallel** with a resistance **R_N**, where I_N is the short-circuit current through the load terminals and R_N is the resistance seen from the load terminals with all sources replaced by their internal resistances (voltage sources shorted, current sources opened).

Load current $I_L = I_N \times R_N / (R_N + R_L)$

d) Active and reactive power. 2 M

Active (true) power P: The power actually consumed (converted into heat/work) by the resistive part of the circuit. $P = V I \cos\varphi$. Unit: **watt (W)**.

Reactive power Q: The power which flows back and forth between the source and the reactive elements (L, C); it is not consumed. $Q = V I \sin\varphi$. Unit: **volt-ampere reactive (VAR)**.

(Each definition with unit 1 M)

e) Condition for series resonance; resonant frequency. 2 M

Condition: A series R-L-C circuit is at resonance when inductive reactance equals capacitive reactance, $X_L = X_C$, i.e. $2\pi fL = 1/(2\pi fC)$. Then $Z = R$ (minimum), current is maximum and power factor is unity.

Resonant frequency: $f_r = 1 / (2\pi\sqrt{(LC)})$ Hz

f) Z-parameter equations. 2 M

Taking I_1 and I_2 as independent variables:

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

where $Z_{11} = V_1/I_1 |_{I2=0}$, $Z_{12} = V_1/I_2 |_{I1=0}$, $Z_{21} = V_2/I_1 |_{I2=0}$, $Z_{22} = V_2/I_2 |_{I1=0}$ (open-circuit parameters, unit Ω).

g) Pass band, stop band, cut-off frequency. 2 M

- **Pass band:** The range of frequencies which a filter passes to the output with no (or very little) attenuation.
- **Stop band (attenuation band):** The range of frequencies which the filter attenuates (does not allow to pass).
- **Cut-off frequency f_c:** The frequency which separates the pass band and stop band; at f_c the output falls to 0.707 of maximum (−3 dB).

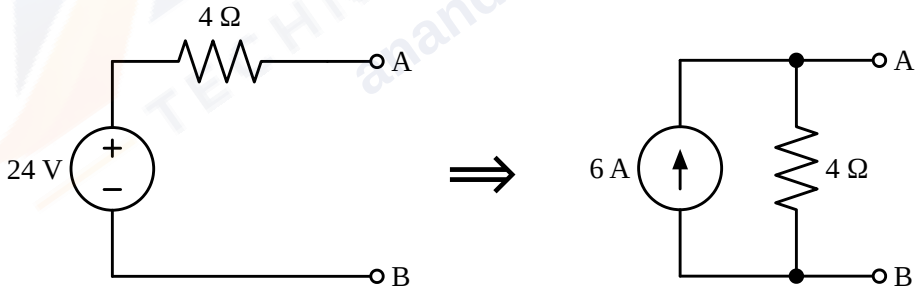
Q.2

a) Convert 24 V, 4 Ω practical voltage source into current source. 4 M

Steps:

1. Find the current source value I = V / r (current that flows when terminals A-B are short-circuited).
2. Keep the same internal resistance r, but connect it in **parallel** with the current source.
3. Direction of the current source arrow is towards the terminal which was connected to the + terminal of the voltage source.
4. Both sources give the same voltage and current to any load connected at A-B.

Given: V = 24 V, r = 4 Ω ⟹ I = V / r = 24 / 4 = **6 A**, parallel resistance = 4 Ω.



(a) Given practical voltage source *(b) Equivalent practical current source*

Check (load R_L = 8 Ω): from (a) I_L = 24/(4+8) = 2 A; from (b) I_L = 6 × 4/(4+8) = 2 A — same.

(Steps 2 M, calculation and diagrams 2 M)

b) Superposition theorem: statement, steps, limitations. 4 M

Statement: In a linear bilateral network containing more than one source, the current through (or voltage across) any element is equal to the algebraic sum of the currents (or voltages) produced by

each source acting alone, all other sources being replaced by their internal resistances.

Steps:

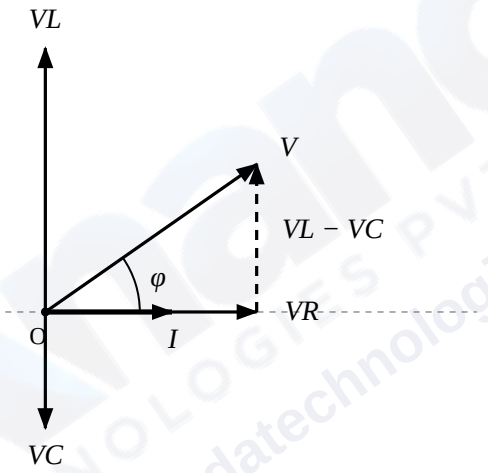
1. Keep one source in the circuit; replace other ideal voltage sources by short circuit and ideal current sources by open circuit (keep internal resistances).
2. Find the current through the required branch (magnitude and direction) due to this source, using series-parallel reduction / current division.
3. Repeat steps 1 and 2 for every source one by one.
4. Add all these currents algebraically (taking directions into account) to get the actual current.

Limitations (any two): i) Applicable only to linear networks. ii) Cannot be used directly to calculate power (power is not linear, $P = I^2R$). iii) Not applicable to networks with only one source / unilateral (non-linear) elements like diodes.

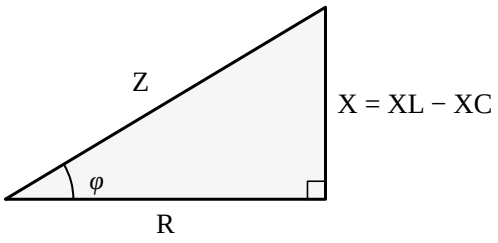
(Statement 1 M, steps 2 M, limitations 1 M)

c) Series RLC ($X_L > X_C$): phasor diagram, impedance and power triangles. 4 M

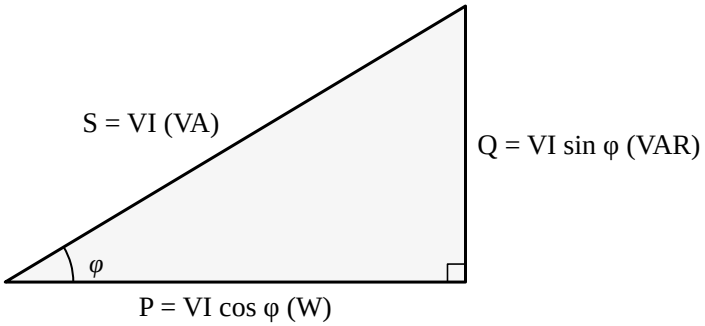
When $X_L > X_C$, $V_L > V_C$; the circuit behaves as an **inductive** circuit and current I **lags** the supply voltage V by angle φ. Current I is taken as reference (common in series circuit).



Phasor diagram (XL > XC, current lags V by φ)



Impedance triangle



Power triangle

$Z = \sqrt{(R^2 + (X_L - X_C)^2)}$, $\varphi = \tan^{-1}((X_L - X_C)/R)$, power factor $\cos\varphi = R/Z$ (lagging).

(Phasor diagram 2 M, each triangle 1 M)

d) Coil 30 Ω, 0.12 H on 230 V, 50 Hz. 4 M

i) $X_L = 2\pi fL = 2\pi \times 50 \times 0.12 =$ **37.699 Ω**

ii) $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(30^2 + 37.699^2)} =$ **48.179 Ω**

iii) $I = V / Z = 230 / 48.179 =$ **4.774 A**

iv) Power factor $\cos\varphi = R / Z = 30 / 48.179 =$ **0.6227 (lagging)**, $\varphi = 51.49°$

Power consumed $P = V I \cos\varphi = 230 \times 4.774 \times 0.6227 =$ **683.69 W** (check: $I^2R = 683.69$ W)

(Each quantity 1 M)

Q.3

a) Equivalent resistance between A and B (Fig. No. 1). 4 M

Start reducing from the end farthest from terminals A-B:

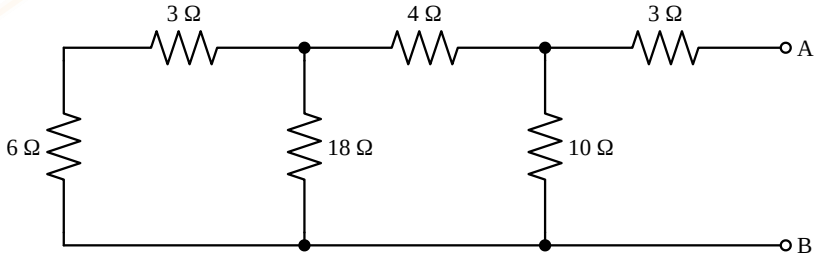
Step 1: 6 Ω and 3 Ω are in series → 6 + 3 = 9 Ω

Step 2: 9 Ω is in parallel with 18 Ω → (9 × 18)/(9 + 18) = 6 Ω

Step 3: 6 Ω in series with 4 Ω → 10 Ω

Step 4: 10 Ω in parallel with 10 Ω → (10 × 10)/(10 + 10) = 5 Ω

Step 5: 5 Ω in series with 3 Ω → 8 Ω

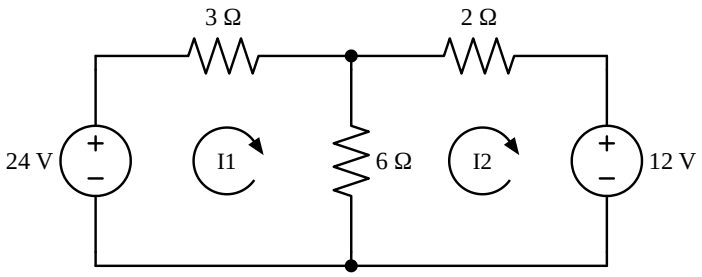


R_{AB} = 8 Ω

(Correct steps 3 M, answer 1 M)

b) Mesh analysis — current through 6 Ω (Fig. No. 2). 4 M

Assume clockwise mesh currents I1 and I2 as shown. The 6 Ω resistor is common to both meshes.



KVL for mesh 1 (Σ IR drops = Σ emfs in direction of I1):

$3I1 + 6(I1 - I2) = 24$

$\Rightarrow 9I1 - 6I2 = 24$ ……(1)

KVL for mesh 2:

$6(I2 - I1) + 2I2 = -12$

$\Rightarrow -6I1 + 8I2 = -12$ ……(2)

Solving (1) and (2) by Cramer's rule:

Writing in matrix form:

$$\begin{vmatrix} 9 & -6 \\ -6 & 8 \end{vmatrix} \times \begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} 24 \\ -12 \end{vmatrix}$$

$\Delta = | 9\ -6\ ;\ -6\ 8 | =$ **36**

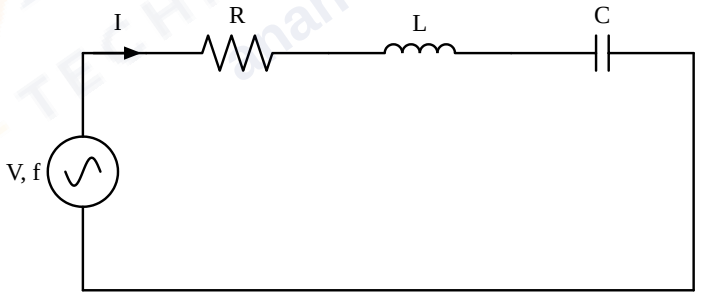
$\Delta 1$ (replace column 1 by constants) $= | 24\ -6\ ;\ -12\ 8 | =$ **120** $\Rightarrow$ $I1 = \Delta 1/\Delta = 120/36 =$ **3.3333 A**

$\Delta 2$ (replace column 2 by constants) $= | 9\ 24\ ;\ -6\ -12 | =$ **36** $\Rightarrow$ $I2 = \Delta 2/\Delta = 36/36 =$ **1 A**

Current through 6 Ω = I1 − I2 = 3.3333 − (1) = **2.3333 A** (downward)

(Mesh equations 2 M, solution 1 M, answer 1 M)

c) Derivation of resonant frequency of series RLC circuit. 4 M



Series R-L-C circuit

Impedance of the series R-L-C circuit:

$Z = R + j(X_L - X_C) = R + j(\omega L - 1/\omega C)$

At resonance the circuit is purely resistive, i.e. the imaginary part (net reactance) is zero:

$X_L = X_C \Rightarrow \omega_r L = 1/(\omega_r C)$

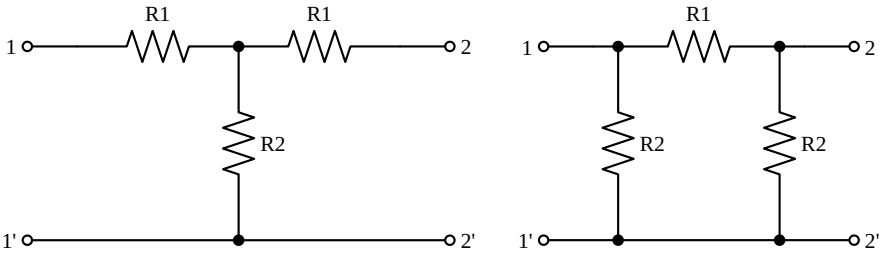
$\omega_r^2 = 1/(LC) \Rightarrow \omega_r = 1/\sqrt{(LC)}$

Since $\omega_r = 2\pi f_r$: **$f_r = 1 / (2\pi\sqrt{(LC)})$ Hz**

At f_r: Z = R (minimum), I = V/R (maximum), power factor = 1.

(Circuit 1 M, derivation 3 M)

d) Compare T and π attenuators. 4 M



(a) Symmetrical T attenuator *(b) Symmetrical π attenuator*

Point	T attenuator	π attenuator
Structure	Two series arms and one shunt arm (looks like letter T).	One series arm and two shunt arms (looks like letter π).
No. of resistors in series path	Two resistors in the series path.	One resistor in the series path.
Design formulae	R1 = R0 (N − 1)/(N + 1), R2 = 2 R0 N/(N² − 1)	R1 = R0 (N² − 1)/(2N), R2 = R0 (N + 1)/(N − 1)
Equivalent	Equivalent to star (Y) network.	Equivalent to delta (Δ) network.
Balanced form	Balanced form is the H attenuator.	Balanced form is the O attenuator.

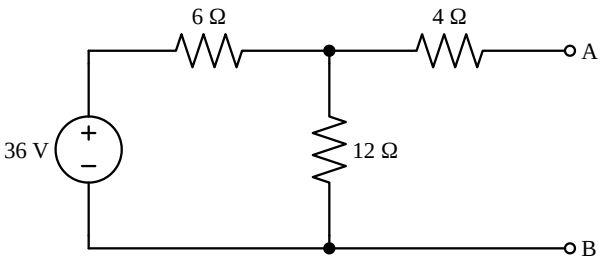
(R_0 = characteristic resistance, N = attenuation ratio V_{in}/V_{out}.)

(Any four points, 1 M each)

Q.4

a) Thevenin's theorem — current through R_L = 5 Ω (Fig. No. 3). 4 M

Step 1 — V_{th}: Remove R_L and find open-circuit voltage across A-B. No current flows in 4 Ω (open), so V_{AB} = voltage across 12 Ω.

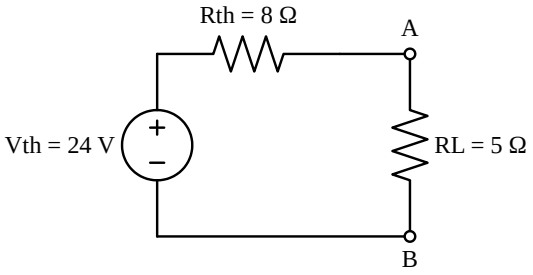


Current in loop = 36 / (6 + 12) = 2 A; V_{th} = 2 × 12 = **24 V**

Step 2 — R_{th}: Replace 36 V source by short circuit and find resistance seen from A-B:

R_{th} = (6 ∥ 12) + 4 = (6 × 12)/(6 + 12) + 4 = 4 + 4 = **8 Ω**

Step 3 — Thevenin equivalent circuit and load current:



Thevenin's equivalent circuit

$I_L = V_{th} / (R_{th} + R_L) = 24 / (8 + 5) =$ **1.8462 A**

(Vth 1 M, Rth 1 M, equivalent circuit 1 M, IL 1 M)

b) Nodal analysis (Fig. No. 4). 4 M

Bottom node is taken as reference (ground). Unknown node voltages are V1 and V2. Writing KCL (sum of currents leaving the node = 0):

Node 1:

$(V1 - 24)/4 + V1/8 + (V1 - V2)/2 = 0$

$\Longrightarrow$ (clearing fractions) $7V1 - 4V2 = 48$ ……(1)

Node 2 (3 A enters node 2):

$V2/6 - 3 + (V2 - V1)/2 = 0$

$\Longrightarrow -3V1 + 4V2 = 18$ ……(2)

Writing in matrix form:

$$\begin{vmatrix} 7 & -4 \\ -3 & 4 \end{vmatrix} \times \begin{vmatrix} V1 \\ V2 \end{vmatrix} = \begin{vmatrix} 48 \\ 18 \end{vmatrix}$$

$\Delta = | 7 \ -4 \ ; \ -3 \ 4 | =$ **16**

$\Delta 1$ (replace column 1 by constants) $= | 48 \ -4 \ ; \ 18 \ 4 | =$ **264** $\Longrightarrow$ $V1 = \Delta 1/\Delta = 264/16 =$ **16.5 V**

$\Delta 2$ (replace column 2 by constants) $= | 7 \ 48 \ ; \ -3 \ 18 | =$ **270** $\Longrightarrow$ $V2 = \Delta 2/\Delta = 270/16 =$ **16.875 V**

Current through 6 Ω = V2 / 6 = 16.875 / 6 = **2.8125 A**

(Node equations 2 M, solution 2 M)

c) Z-parameters of T network (Fig. No. 5). 4 M

(i) Port 2 open ($I_2 = 0$): current I_1 flows through 4 Ω and 10 Ω.

$V_1 = I_1(4 + 10) \Longrightarrow Z_{11} = V_1/I_1 =$ **14 Ω**

V_2 = voltage across 10 Ω = 10 $I_1 \Longrightarrow Z_{21} = V_2/I_1 =$ **10 Ω**

(ii) Port 1 open ($I_1 = 0$): current I_2 flows through 6 Ω and 10 Ω.

$V_2 = I_2(6 + 10) \Longrightarrow Z_{22} =$ **16 Ω**

$V_1 = 10\ I_2 \Longrightarrow Z_{12} =$ **10 Ω**

$$[Z] = \begin{vmatrix} 14 & 10 \\ & \end{vmatrix} \ \Omega$$

$$\begin{vmatrix} 10 & 16 \end{vmatrix}$$

Since $Z_{12} = Z_{21} = 10\ \Omega$, the network is **reciprocal**. Since Z_{11} (14 Ω) ≠ Z_{22} (16 Ω), it is **not symmetrical**.

(Each parameter 1 M (reciprocity/symmetry comments included))

d) Series RC: R = 20 Ω, C = 100 µF, 230 V, 50 Hz. 4 M

i) $X_C = 1/(2\pi fC) = 1/(2\pi \times 50 \times 100 \times 10^{-6}) =$ **31.831 Ω**

ii) $Z = \sqrt{(R^2 + X_C^2)} = \sqrt{(20^2 + 31.831^2)} =$ **37.593 Ω**

iii) $I = V/Z = 230 / 37.593 =$ **6.118 A**

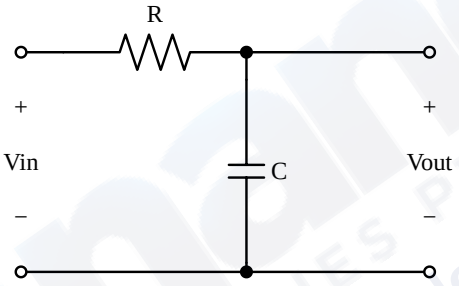
iv) $\cos\varphi = R/Z = 20 / 37.593 =$ **0.532 (leading)**, $\varphi = 57.86°$

v) $P = V I \cos\varphi = 230 \times 6.118 \times 0.532 =$ **748.65 W** ($= I^2R = 748.65$ W)

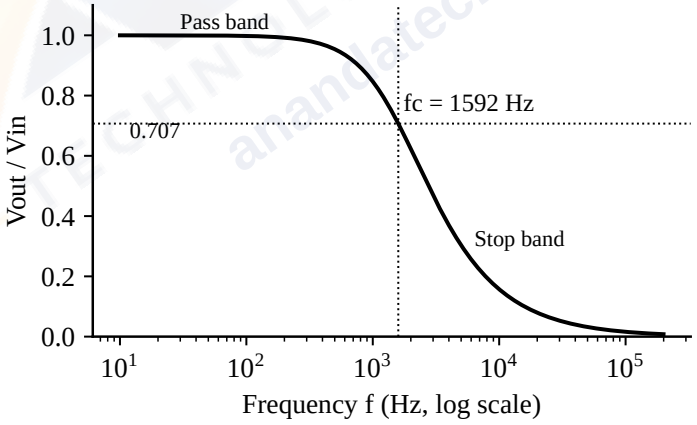
(Each 1 M (any four))

e) RC low pass filter, response and cut-off frequency. 4 M

A **low pass filter** passes all frequencies from 0 (dc) up to the cut-off frequency f_c and attenuates frequencies above f_c. In the RC LPF the output is taken across C; since $X_C = 1/(2\pi fC)$ decreases with frequency, high frequencies are bypassed.



RC low pass filter



Frequency response of RC LPF (R = 1 kΩ, C = 0.1 µF)

Cut-off frequency: at f_c, $X_C = R \Rightarrow$ **$f_c = 1/(2\pi RC)$**

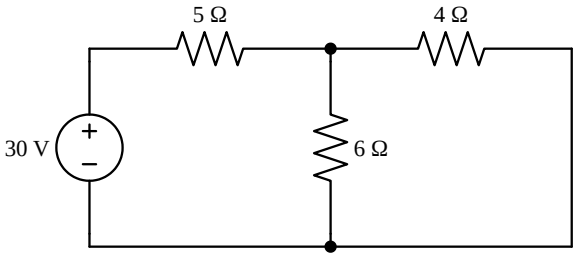
$f_c = 1/(2\pi \times 1000 \times 0.1 \times 10^{-6}) =$ **1591.5 Hz ≈ 1.592 kHz**

(Circuit 1 M, response 1 M, expression 1 M, calculation 1 M)

Q.5

a) Superposition theorem — current through 6 Ω (Fig. No. 6). 6 M

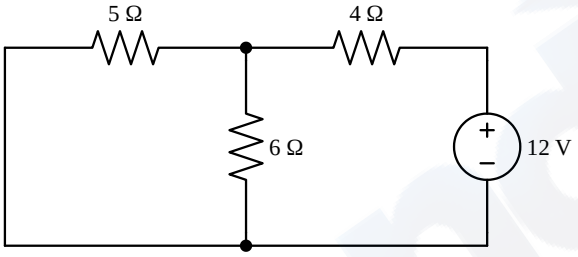
Case 1: 30 V source acting alone (12 V source replaced by short circuit):



$R_T = 5 + (6 \parallel 4) = 5 + 2.4 = 7.4\ \Omega$; $I_T = 30 / 7.4 = 4.0541$ A

By current division, I' (6 Ω, downward) = $I_T \times 4/(6 + 4) = 4.0541 \times 0.4 =$ **1.6216 A** ↓

Case 2: 12 V source acting alone (30 V source replaced by short circuit):



$R_T = 4 + (6 \parallel 5) = 4 + 2.7273 = 6.7273\ \Omega$; $I_T = 12 / 6.7273 = 1.7838$ A

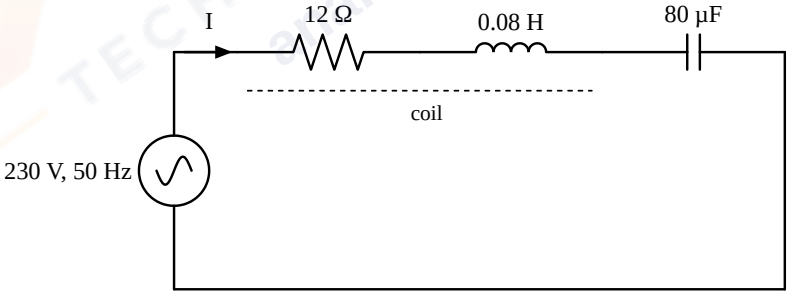
I" (6 Ω, downward) = $I_T \times 5/(6 + 5) =$ **0.8108 A** ↓

Total current: both components flow downward through 6 Ω, so

I = I' + I" = 1.6216 + 0.8108 = **2.4324 A** (downward)

(Case 1: 2.5 M, Case 2: 2.5 M, final 1 M)

b) Coil (12 Ω, 0.08 H) in series with 80 µF on 230 V, 50 Hz. 6 M



$X_L = 2\pi fL = 2\pi \times 50 \times 0.08 = 25.133\ \Omega$; $X_C = 1/(2\pi fC) = 1/(2\pi \times 50 \times 80 \times 10^{-6}) = 39.789\ \Omega$

i) $Z = \sqrt{(R^2 + (X_L - X_C)^2)} = \sqrt{(12^2 + (25.133 - 39.789)^2)} = \sqrt{(12^2 + (-14.656)^2)} =$ **18.942 Ω**

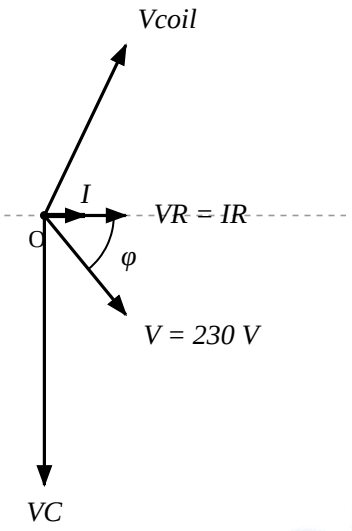
ii) I = V/Z = 230 / 18.942 = **12.142 A**

iii) cos φ = R/Z = 12 / 18.942 = **0.6335 leading** ($X_C > X_L$, circuit is capacitive), φ = 50.69°

iv) Coil impedance $Z_{coil} = \sqrt{(12^2 + 25.133^2)} = 27.851\ \Omega$; $V_{coil} = I \times Z_{coil} = 12.142 \times 27.851 =$ **338.17 V** (leads I by 64.48°)

$V_C = I \times X_C = 12.142 \times 39.789 =$ **483.13 V** (lags I by 90°)

(Note: V_{coil} and V_C are each greater than the supply voltage — this is possible in a series AC circuit because they are not in phase; their phasor sum is 230 V.)



Phasor diagram (I as reference; I leads V by φ)

(Z 1 M, I 1 M, pf 1 M, voltages 1.5 M, phasor diagram 1.5 M)

c) Two-port interconnections. 6 M

i) Cascade (tandem) connection: The output port of the first network N1 is connected to the input port of the second network N2. Overall **ABCD (transmission) parameter** matrix is the product: $[T] = [T_1] \times [T_2]$.

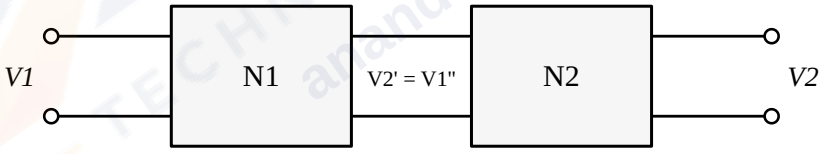


Fig. (i) Cascade connection

ii) Series connection: Input ports are in series and output ports are in series; the same current flows through both networks at each port. Overall **Z-parameters** are added: $[Z] = [Z_1] + [Z_2]$.

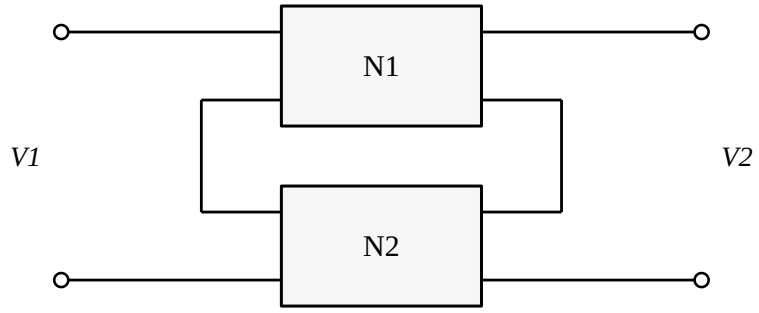


Fig. (ii) Series connection

iii) Parallel connection: Input ports are in parallel and output ports are in parallel; the same voltage appears across both networks at each port. Overall **Y-parameters** are added: $[Y] = [Y_1] + [Y_2]$.

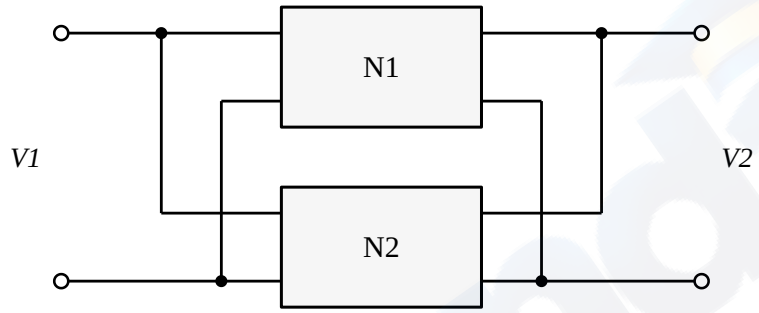


Fig. (iii) Parallel connection

(Each diagram 1 M, each parameter relation 1 M)

Q.6

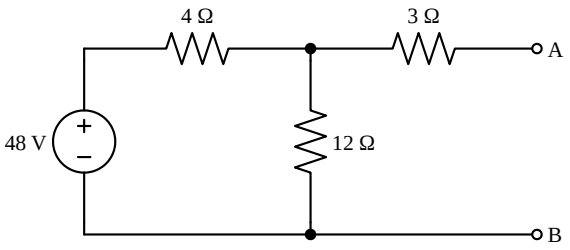
a) Maximum power transfer (Fig. No. 7). 6 M

By maximum power transfer theorem, R_L receives maximum power when $R_L = R_{th}$ (Thevenin resistance seen from load terminals).

Step 1 — R_{th}: Remove R_L, short the 48 V source:

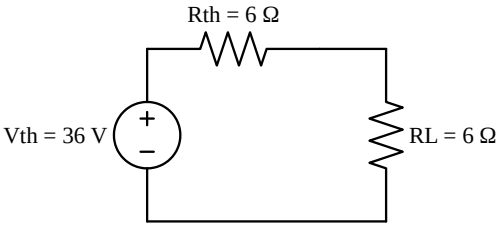
$R_{th} = (4 \parallel 12) + 3 = 3 + 3 =$ **6 Ω**

Step 2 — V_{th}: With R_L removed, no current in 3 Ω; V_{th} = voltage across 12 Ω:



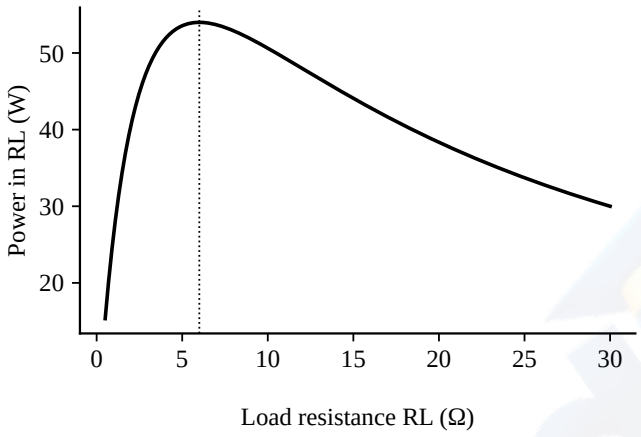
$V_{th} = 48 \times 12/(4 + 12) =$ **36 V**

Step 3: For maximum power, $\mathbf{R_L = R_{th} = 6\ \Omega}$.



Thevenin equivalent with matched load

$P_{max} = V_{th}^2 / (4 R_{th}) = 36^2 / (4 \times 6) =$ **54 W**



Power vs RL — peak at RL = Rth (computed)

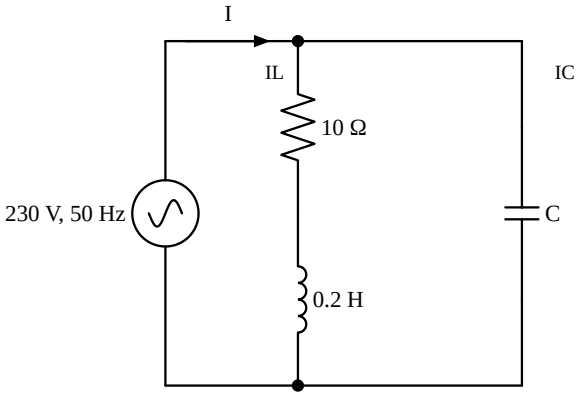
(Rth 2 M, Vth 2 M, RL and Pmax 2 M)

b) Compare series and parallel resonant circuits. 6 M

Parameter	Series resonant circuit	Parallel resonant circuit
i) Resonant frequency	$f_r = 1/(2\pi\sqrt{(LC)})$	$f_r = (1/2\pi)\sqrt{(1/(LC) - R^2/L^2)}$
ii) Impedance at resonance	Minimum, Z = R	Maximum, $Z_D = L/(CR)$ (dynamic impedance)
iii) Current at resonance	Maximum, I = V/R	Minimum, $I = V/Z_D$
iv) Magnification	Voltage magnification: $V_L = V_C = Q$ V	Current magnification: $I_L \approx I_C = Q$ I
v) Power factor	Unity	Unity
vi) Application	Acceptor circuit — used in tuning circuits of radio receivers, filters to accept a frequency	Rejector circuit — used in tank circuits of oscillators, IF amplifiers, wave traps to reject a frequency

(Each point 1 M)

c) Parallel resonance: coil 10 Ω, 0.2 H with variable C, 230 V, 50 Hz. 6 M



For a coil (R, L) in parallel with C, resonant frequency:

$f_r = (1/2\pi)\sqrt{(1/(LC) - R^2/L^2)} \implies C = L / (R^2 + \omega^2 L^2)$

i) $\omega = 2\pi \times 50 = 314.159$ rad/s, $\omega L = 62.832\ \Omega$; $C = 0.2 / (10^2 + 62.832^2) = 0.2 / 4047.84 =$ **49.409 μF**

ii) Q-factor of coil = $\omega_r L / R = 62.832 / 10 =$ **6.283**

iii) Dynamic impedance $Z_D = L/(CR) = 0.2 / (49.409 \times 10^{-6} \times 10) =$ **404.78 Ω**

iv) Supply current at resonance $I = V / Z_D = 230 / 404.78 =$ **0.5682 A** (in phase with V)

(For reference: coil current = $230/\sqrt{(10^2 + 62.832^2)} = 3.615$ A, capacitor current = $230 \times \omega C = 3.57$ A — both much larger than the supply current, showing current magnification.)

(C 2 M, Q 1 M, ZD 2 M, I 1 M)

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