

Theory of Structure (315313)

MSBTE K-Scheme · Civil Engineering (CE) Semester 5 · Model Question Paper — Set A · Model Answers

Sign conventions used: sagging BM positive (+), hogging BM negative (–) in diagrams; support moments in three-moment equations are written as hogging magnitudes; in moment distribution, clockwise end moments are positive. Units: kN, m unless stated.

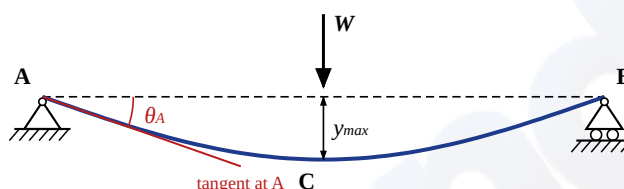
Q.1 Attempt any FIVE of the following: (10 M)

a) Slope and deflection of a beam

2 M

Slope: The slope at any section of a deflected beam is the **angle (in radians) made by the tangent to the elastic curve** at that section with the original (undeflected) axis of the beam. $\theta = dy/dx$.

Deflection: The **vertical displacement** of any point on the axis of the beam from its original position, measured perpendicular to the original axis, is called deflection (y). Unit: mm.



Dashed: original axis · Blue: elastic curve (deflected axis)

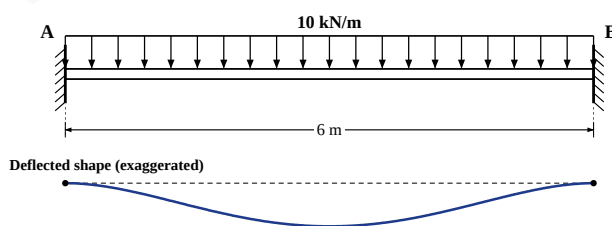
Fig.: Slope (θ) and deflection (y) of a simply supported beam

(Each definition $\frac{1}{2}$ M, sketch 1 M)

b) Fixed beam

2 M

Fixed beam: A beam whose **both ends are rigidly fixed** (built-in) so that at the supports the **slope and deflection are zero** is called a fixed beam (also built-in or encastré beam). Fixing moments (hogging) develop at the supports. Example: RCC beam cast monolithic with columns.



Fixed beam: slope = 0 and deflection = 0 at A and B

(Definition 2 M)

c) Stiffness factor and distribution factor

2 M

Stiffness factor: The moment required to produce a **unit rotation** at one end of a member (near end) when the far end is fixed is called the stiffness factor. $k = 4EI/L$ (far end fixed) and $k = 3EI/L$

(far end hinged).

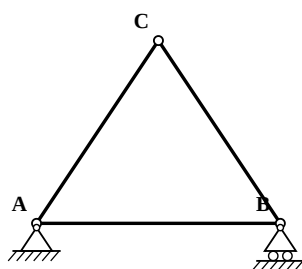
Distribution factor: The ratio in which an unbalanced moment at a joint is distributed to the members meeting at that joint. D.F. = k of member / Σk of all members at the joint. Σ D.F. at a joint = 1.

(Each definition 1 M)

d) Perfect frame

2 M

Perfect frame: A frame which has just sufficient number of members to remain in equilibrium without any distortion when loaded. It satisfies $m = 2j - 3$ (m = members, j = joints). Example: a triangle ($m = 3$, $j = 3$).



Perfect frame: $m = 3 = 2(3) - 3$

(Definition 1 M, sketch 1 M)

e) Effects of continuity

2 M

Effect of continuity on a continuous beam:

1. **Hogging (negative) moments** develop over the intermediate supports.
2. The **sagging moments in the spans are reduced** compared to separate simply supported spans, so the maximum BM and section required are smaller.
3. The **deflections are reduced**; the elastic curve is continuous over the supports (no kink).
4. Reactions at intermediate supports increase and those at end supports decrease.

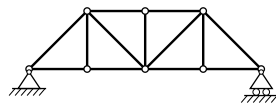
(Any two effects 1 M each)

f) Types of trusses

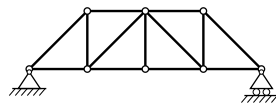
2 M

Types of trusses (any four):

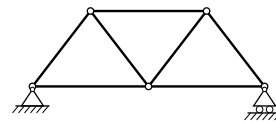
1. Pratt truss
2. Howe truss
3. Warren truss
4. King post truss
5. Queen post truss
6. Fink (French) truss



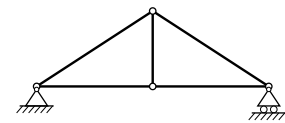
Pratt truss



Howe truss



Warren truss



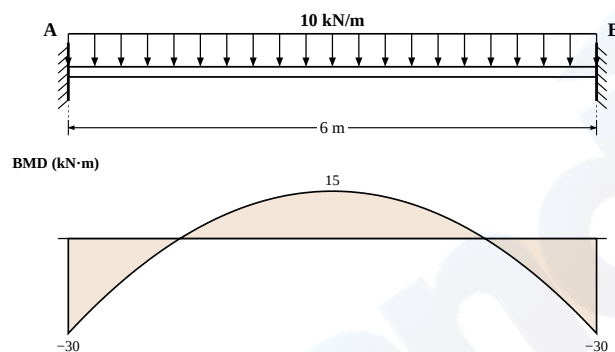
King post truss

(Any four, $\frac{1}{2}$ M each)

g) Point of contraflexure

2 M

Point of contraflexure: The point along a beam where the **bending moment changes its sign** (from sagging to hogging or vice versa) and the BM is **zero** is called the point of contraflexure (point of inflection). Example: a fixed beam with UDL has two points of contraflexure at $0.211L$ from each end.



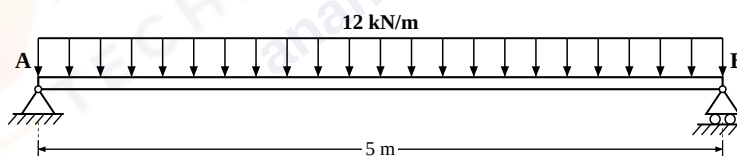
BMD of fixed beam (UDL 10 kN/m, 6 m): points of contraflexure at 1.27 m from each end

(Definition 2 M)

Q.2 Attempt any THREE of the following: (12 M)

a) SS beam with UDL — standard formulae

4 M



Simply supported beam with UDL

Given: $L = 5 \text{ m} = 5000 \text{ mm}$, $w = 12 \text{ kN/m} = 12 \text{ N/mm}$, $E = 200,000 \text{ N/mm}^2$, $I = 8 \times 10^7 \text{ mm}^4$

Flexural rigidity $EI = 200,000 \times 8 \times 10^7 = 1.6 \times 10^{13} \text{ N}\cdot\text{mm}^2 = \mathbf{16000 \text{ kN}\cdot\text{m}^2}$

Standard formulae (SS beam with UDL on whole span):

Maximum slope (at supports) $\theta_A = \theta_B = wL^3 / 24EI$; Maximum deflection (at mid-span) $y_{\max} = 5wL^4 / 384EI$

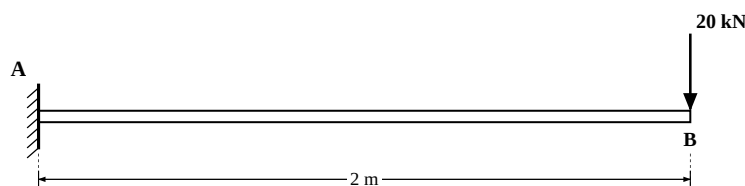
$$\theta_{\max} = 12 \times 5^3 / (24 \times 16000) = \mathbf{0.00391 \text{ rad}} = 0.224^\circ$$

$$y_{\max} = 5 \times 12 \times 5^4 / (384 \times 16000) = 0.006104 \text{ m} = \mathbf{6.10 \text{ mm}} \text{ (downward, at centre)}$$

(Formulae 1 M, EI 1 M, slope 1 M, deflection 1 M)

b) Cantilever with end point load

4 M



Cantilever AB (A fixed, B free)

Given: $L = 2 \text{ m}$, $W = 20 \text{ kN}$ at free end, $E = 20,000 \text{ N/mm}^2$

$$I = bd^3/12 = 150 \times 300^3 / 12 = 3.375 \times 10^8 \text{ mm}^4$$

$$EI = 20,000 \times 3.375 \times 10^8 = 6.75 \times 10^{12} \text{ N}\cdot\text{mm}^2 = 6750 \text{ kN}\cdot\text{m}^2$$

Load case	Slope at B	θ (rad)	Deflection at B	y (mm)
Point load W at free end	$WL^2/2EI$	0.00593	$WL^3/3EI$	7.90

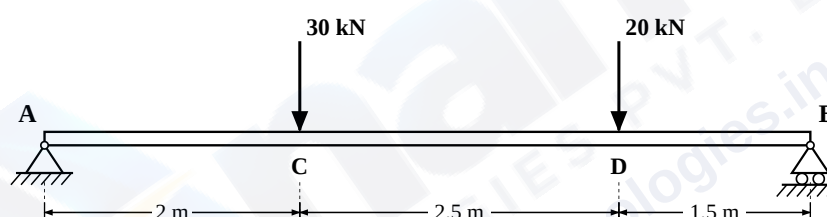
Slope at free end $\theta_B = 0.00593 \text{ rad}$ (0.340°)

Deflection at free end $y_B = 7.90 \text{ mm}$ (downward)

(I and EI 1 M, formulae 1 M, slope 1 M, deflection 1 M)

c) Macaulay's method — deflection under loads

4 M



Step 1 — Reactions: $\Sigma M_A = 0$; $R_B \times 6 = 30 \times 2 + 20 \times 4.5 \rightarrow R_B = 25 \text{ kN}$; $R_A = (\text{total load}) - R_B = 25 \text{ kN}$

Step 2 — Macaulay's equation: Take A as origin and a section X-X at distance x from A in the last segment. (Terms after the vertical bar apply only when the bracket is positive.)

$$EI \frac{d^2y}{dx^2} = M_x = 25x - 30[x - 2] - 20[x - 4.5]$$

$$EI \frac{dy}{dx} = 12.5x^2 - 15[x - 2]^2 - 10[x - 4.5]^2 + C_1 \quad \dots (1)$$

$$EI y = 4.167x^3 - 5[x - 2]^3 - 3.333[x - 4.5]^3 + C_1x + C_2 \quad \dots (2)$$

Step 3 — Boundary conditions: At $x = 0$, $y = 0 \rightarrow$ (first segment of eq. 2) $C_2 = 0$. At $x = 6 \text{ m}$, $y = 0 \rightarrow$ (all terms of eq. 2) $C_1 = -94.792$

Step 4 — Results:

- Deflection at C ($x = 2 \text{ m}$): $EI y = -156.25 \rightarrow y_C = 156.25/EI$ (downward)
- Deflection at D ($x = 4.5 \text{ m}$): $EI y = -125 \rightarrow y_D = 125/EI$ (downward)

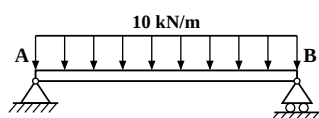
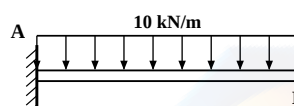
✓ Check: values agree with an independent stiffness-matrix analysis of the same beam.

(Reactions 1 M, equation 1 M, constants 1 M, deflections 1 M)

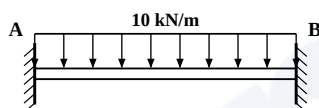
d) Boundary conditions

4 M

Beam	Boundary conditions	Remarks
Simply supported beam (supports A and B)	At $x = 0$: $y = 0$; at $x = L$: $y = 0$	Slope is maximum at supports; at mid-span (symmetrical load) $dy/dx = 0$
Cantilever (fixed at A, free at B)	At fixed end: $y = 0$ and $dy/dx = 0$	At free end: $BM = 0$; maximum y and θ at free end
Fixed beam (both ends fixed)	At $x = 0$ and $x = L$: $y = 0$ and $dy/dx = 0$	Fixing moments at both ends



SS beam: $y = 0$ at A and B; slope $\neq 0$ at supports Cantilever: at fixed end A, $y = 0$ and $dy/dx = 0$



Fixed beam: $y = 0$ and $dy/dx = 0$ at A and B

(SS 1 M, cantilever 1 M, fixed 1 M, sketches 1 M)

Q.3 Attempt any THREE of the following: (12 M)

a) Advantages and disadvantages of fixed beam

4 M

Advantages of fixed beam over SS beam:

1. Maximum bending moment is less than that of a SS beam of the same span and load (e.g. $wL^2/12$ instead of $wL^2/8$), so a **lighter / smaller section** is required — economical.
2. **Deflection is much smaller** ($wL^4/384EI$ against $5wL^4/384EI$ for UDL — one-fifth).

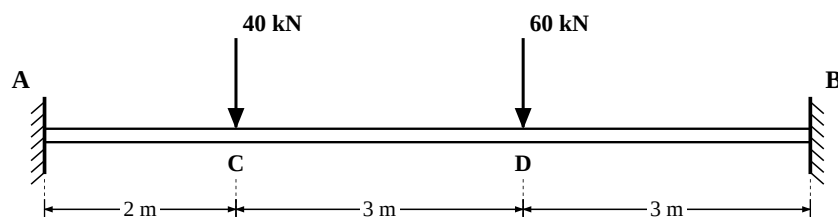
Disadvantages:

1. Fixing moments are produced at the supports, so **supports (walls/columns) must be strong** and are costlier.
2. **Sinking of a support** or rotation of a support causes additional large moments.

(Each point $\frac{1}{2}$ M)

b) Fixed beam — FEM and BMD

4 M



Step 1 — Fixed end moments (hogging, by standard formulae):

Load	M_A (kN·m)	M_B (kN·m)
Point load 40 kN at 2 m from A ($a = 2$, $b = 6$)	$Wab^2/L^2 = 40 \times 2 \times 6^2/8^2 = 45$	$Wa^2b/L^2 = 40 \times 2^2 \times 6/8^2 = 15$
Point load 60 kN at 5 m from A ($a = 5$, $b = 3$)	$Wab^2/L^2 = 60 \times 5 \times 3^2/8^2 = 42.19$	$Wa^2b/L^2 = 60 \times 5^2 \times 3/8^2 = 70.31$
Total	87.19	85.31

Fixed end moments: $M_A = -87.19$ kN·m and $M_B = -85.31$ kN·m (negative = hogging).

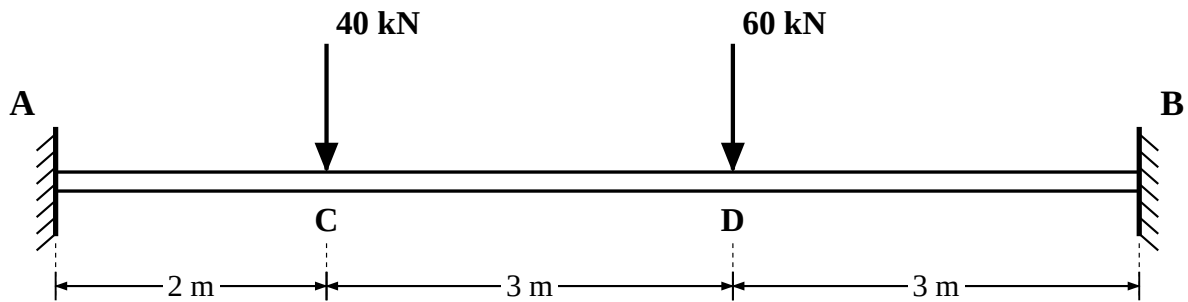
Step 2 — Reactions: $R_A = (\text{free reaction}) + (M_A - M_B)/L = 52.5 + (87.19 - 85.31)/8 = 52.73$ kN;

$R_B = 47.5 - (87.19 - 85.31)/8 = 47.27$ kN

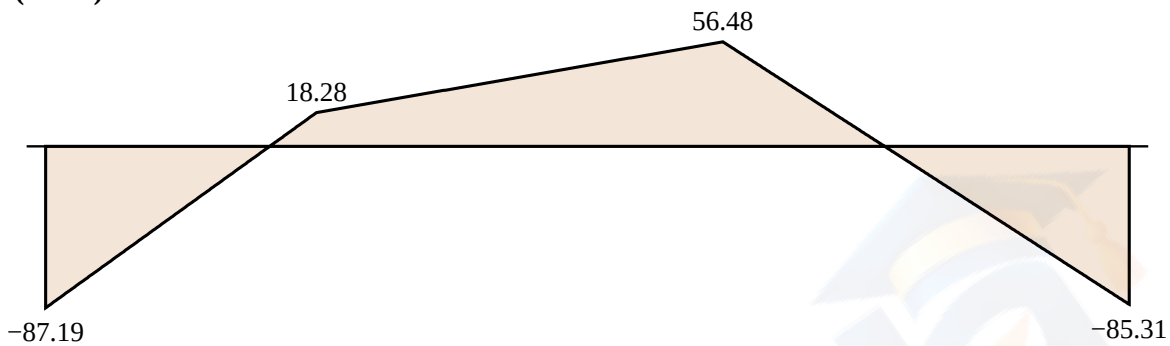
Step 3 — Bending moments (net BM = free BM of SS beam – fixed end moment line):

Section (from A)	Free BM (kN·m)	Fixed-moment line (kN·m)	Net BM (kN·m)
A (0 m)	0	87.19	-87.19
2 m	105	86.72	18.28
5 m	142.5	86.02	56.48
B (8 m)	0	85.31	-85.31

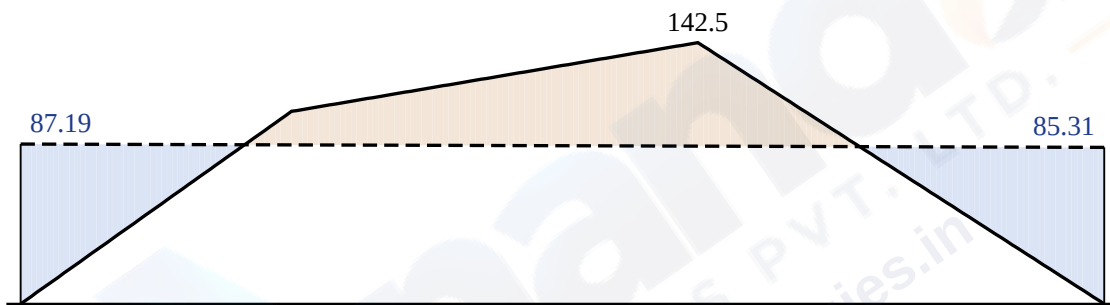
Points of contraflexure (BM = 0): **1.65 m, 6.19 m** from A.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



Deflected shape (exaggerated)



Fixed beam — net BMD, BMD by superposition and deflected shape

✓ Check: stiffness-matrix analysis gives the same end moments (87.19, 85.31 kN·m).

(FEMs 2 M, BM values 1 M, BMD 1 M)

c) Clapeyron's theorem of three moments

4 M

Statement: For a continuous beam with two consecutive spans L_1 and L_2 supported at A, B and C (at the same level), the support moments M_A , M_B and M_C are related as

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6A_1 \bar{x}_1 / L_1 + 6A_2 \bar{x}_2 / L_2$$

where A_1 , A_2 = areas of the free BMDs of spans AB and BC (each treated as simply supported); $\bar{x}_1$ = distance of centroid of A_1 from the left end A; $\bar{x}_2$ = distance of centroid of A_2 from the right end C. Support moments are hogging. If the M.I. differs, each L is replaced by L/I and each $6A\bar{x}/L$ by $6A\bar{x}/(LI)$.

Explanation / use: The equation is written for every set of three consecutive supports; the simultaneous equations give the unknown support moments. For a simply supported end the

moment is zero; for a fixed end an imaginary span of zero length is added. Standard values: UDL $\rightarrow 6A\bar{x}/L = wL^3/4$; central point load $\rightarrow 6A\bar{x}/L = 3WL^2/8$.

Free BMDs (areas A_1, A_2) and support moment line (dashed)

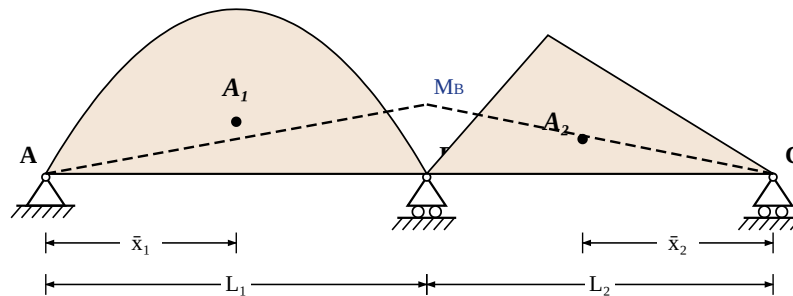


Fig.: Two consecutive spans for Clapeyron's theorem of three moments

(Statement 2 M, explanation with sketch 2 M)

d) Principle of superposition

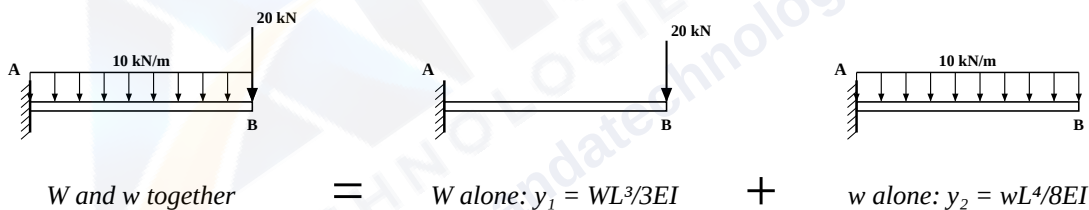
4 M

Principle of superposition: When a beam (linearly elastic, small deflections) is subjected to several loads, the slope, deflection, BM or SF at any section is equal to the **algebraic sum** of the values caused by each load acting separately.

Example: A cantilever of span L carries a point load W at the free end and a UDL w over the whole span. Deflection at the free end:

$$y_B = WL^3/3EI \text{ (due to } W) + wL^4/8EI \text{ (due to } w)$$

e.g. $L = 3 \text{ m}$, $W = 20 \text{ kN}$, $w = 10 \text{ kN/m}$: $y_B = 20 \times 27/3EI + 10 \times 81/8EI = 180/EI + 101.25/EI = 281.25/EI$.

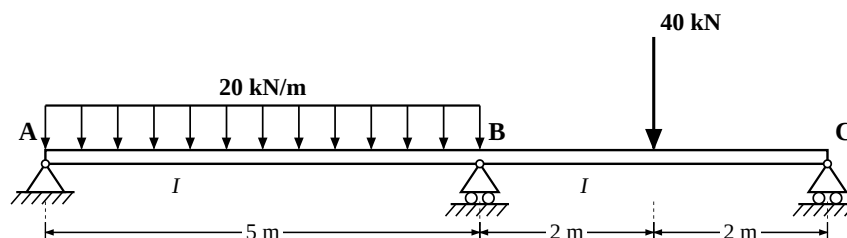


(Statement 2 M, example 2 M)

Q.4 Attempt any THREE of the following: (12 M)

a) Three moment theorem — support moment at B

4 M



Step 1 — Free bending moments and $6A\bar{x}/L$ terms (each span treated as simply supported). For UDL: $6A\bar{x}/L = wL^3/4$; for point load W at distance a from the left end: $6A\bar{x}/L = Wab(L+a)/L$ about

the left end and $Wab(L + b)/L$ about the right end.

Span	L (m)	Max free BM (kN·m)	Area A	$\bar{x}$ from left (m)	$\bar{x}$ from right (m)	$6A\bar{x}_{\text{left}}/L$	$6A\bar{x}_{\text{right}}/L$
AB	5	$wL^2/8 = 20 \times 5^2/8 = 62.5$	208.33	2.5	2.5	625	625
BC	4	$Wab/L = 40 \times 2 \times 2/4 = 40$	80	2	2	240	240

Step 2 — Clapeyron's equations (support moments M taken hogging positive):

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = 6A_1\bar{x}_1/L_1 + 6A_2\bar{x}_2/L_2$$

($\bar{x}_1$ measured from the left support of the left span, $\bar{x}_2$ from the right support of the right span.)

End A is simply supported, so $M_A = 0$. End C is simply supported, so $M_C = 0$.

For supports **A, B, C**: $18 M_B = 865 \dots (1)$

Solving: **$M_B = 48.06 \text{ kN·m}$** (hogging).

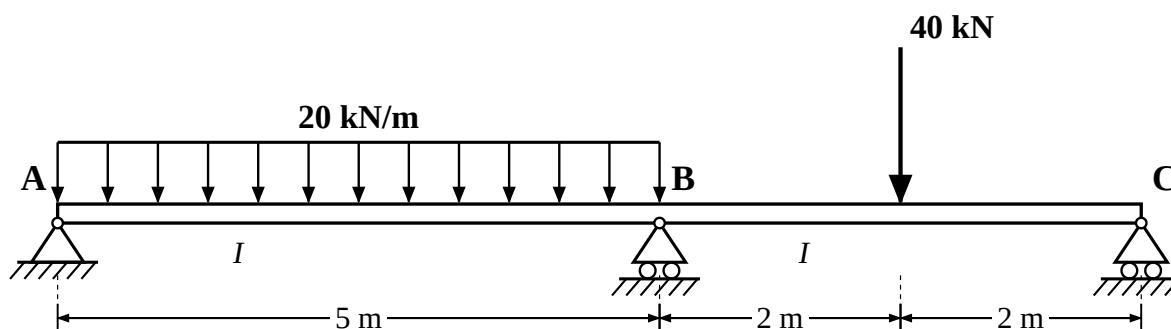
Step 3 — Reactions, SFD and BMD

Support reactions (span-wise: free reaction $\pm (M_{\text{left}} - M_{\text{right}})/L$):

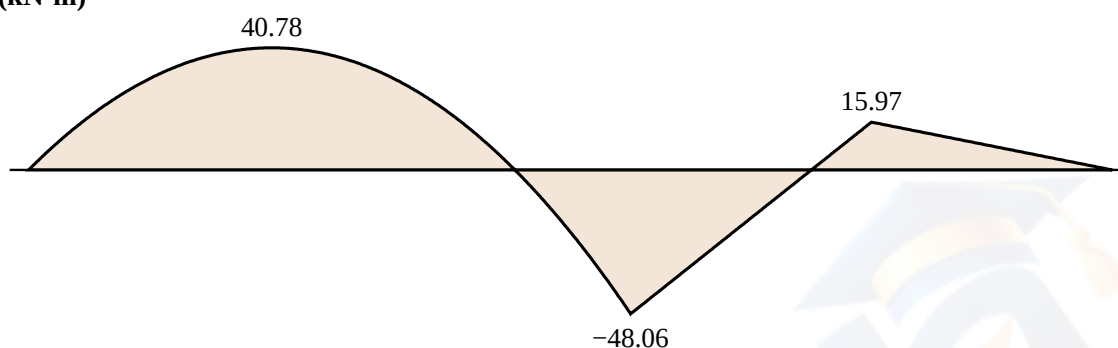
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (5 m)	50 / 50	$(0 - 48.06)/5 = -9.61$	40.39	59.61
BC (4 m)	20 / 20	$(48.06 - 0)/4 = 12.01$	32.01	7.99

$R_A = 40.39 \text{ kN}$; $R_B = 91.62 \text{ kN}$; $R_C = 7.99 \text{ kN}$ ($\Sigma R = 140 \text{ kN} = \text{total load } \checkmark$)

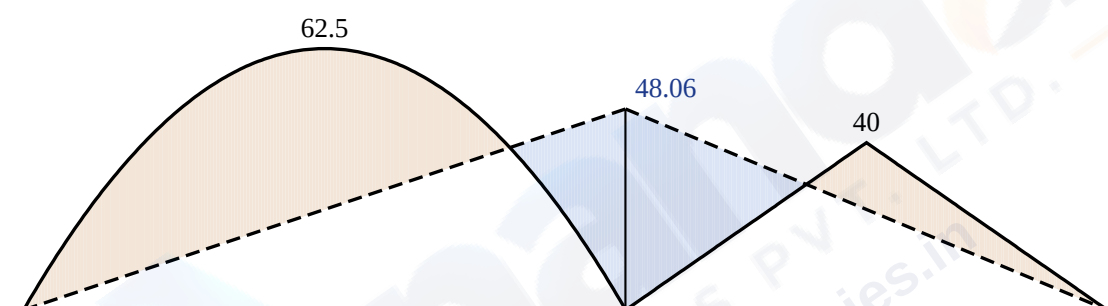
Maximum sagging moments — span AB: **40.78 kN·m** at 2.02 m from A; span BC: **15.97 kN·m** at 2 m from B.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



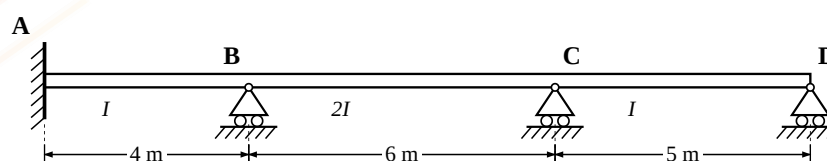
BMD (net) and BMD by superposition (+ sagging, - hogging)

✓ Check: support moments and reactions agree with an independent stiffness-matrix (and moment distribution) solution.

($6A\bar{x}/L$ terms 1 M, equation and M_B 2 M, BMD 1 M)

b) Distribution factors for beam ABCD

4 M



Relative stiffness $k = I/L$ for members whose far end is fixed or continuous; for member CD the far end D is simply supported, so $k_{CD} = (3/4) \cdot I/L$ (modified stiffness). At the fixed end A, $DF = 0$; at the simple end D, $DF = 1$.

Joint	Member	Relative stiffness k	Σk	Distribution factor
A	AB	$I/4 = 0.25I$	0.25I	0 (fixed end)
B	BA	$I/4 = 0.25I$	0.5833I	$0.25/0.5833 = 0.429$

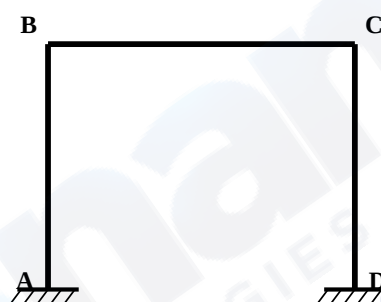
	BC	$2I/6 = 0.3333I$		$0.3333/0.5833 = \mathbf{0.571}$
C	CB	$2I/6 = 0.3333I$	$0.4833I$	$0.3333/0.4833 = \mathbf{0.69}$
	CD	$(3/4) \cdot I/5 = 0.15I$		$0.15/0.4833 = \mathbf{0.31}$
D	DC	$I/5 = 0.2I$	$0.2I$	1 (simple end)

(Stiffnesses $2M$, $DFs\ 2M$)

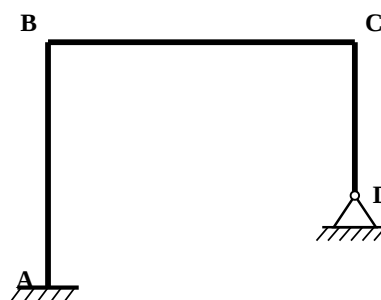
c) Symmetrical vs unsymmetrical portal frames

4 M

Symmetrical portal frame	Unsymmetrical portal frame
Frame and loading are symmetrical about the vertical centre line.	Frame or loading (or both) is not symmetrical.
Columns have equal height and equal M.I.; both bases have same end conditions.	Columns of unequal height / M.I. or different end conditions.
No sway (lateral displacement) occurs under vertical loads.	The frame sways sideways; sway moments must be considered.
Analysis is simple (non-sway analysis; half frame can be used).	Analysis is more complex (sway analysis required).



(a) Symmetrical portal frame: equal columns (h , I), same end conditions, symmetrical load



(b) Unsymmetrical portal frame: unequal columns / different supports / unsymmetrical load — frame sways

(Any four points 1 M each (sketches included))

d) Methods of analysis of frames — method of sections

4 M

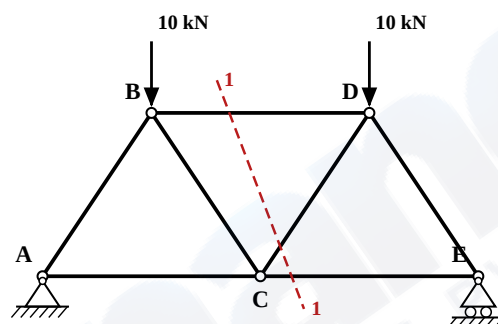
Methods of analysis of a perfect frame (truss):

1. **Analytical methods** — (a) method of joints, (b) method of sections (method of moments).
2. **Graphical method** — force (Maxwell) diagram using Bow's notation.

Method of sections:

1. Find the support reactions.
2. Pass a section line cutting **not more than three members** (with unknown forces) including the required member, dividing the frame into two parts.
3. Consider either part; the forces in the cut members act as external forces on it (assumed tensile).
4. Take moments about the point where two of the cut members meet, so that only one unknown appears; use $\Sigma F_y = 0$ when the other two are parallel.
5. Solve; positive value means tension.

It is useful when forces in only a few members (e.g. in the middle) are required.



Section 1-1 cuts three members

(Methods 1 M, explanation 3 M)

e) Imaginary span

4 M

The theorem of three moments is applicable to three consecutive **simple** supports. When an end of a continuous beam is **fixed**, the theorem cannot be applied directly at that end.

- The fixed end A is replaced by an **imaginary span A_0A of zero length** ($L_0 = 0$) beyond the fixed end, with a simple support at A_0 .
- The imaginary span carries no load, so $A_0 = 0$, and $M_{A_0} = 0$. It is treated as infinitely stiff ($I = \infty$), so the slope at A remains zero — the same condition as a fixed end.
- The theorem is then applied to supports A_0, A, B : $M_{A_0}(0) + 2M_A(0 + L_1) + M_B L_1 = 0 + 6A_1 \bar{x}_1 / L_1$, i.e. $2M_A L_1 + M_B L_1 = 6A_1 \bar{x}_1 / L_1$ ($\bar{x}_1$ from B).

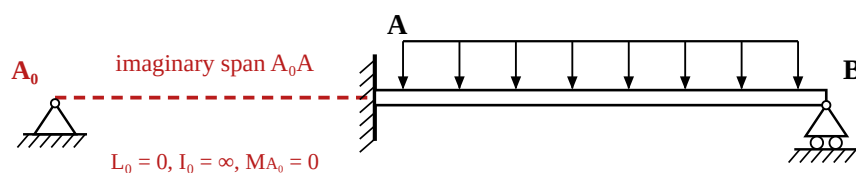


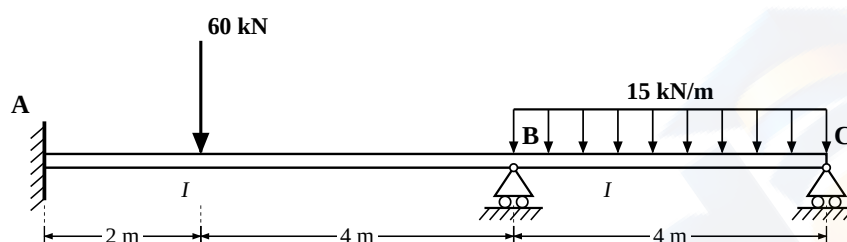
Fig.: Fixed end A replaced by a simple support A_0 with an imaginary span of zero length

(Explanation 2 M, sketch 1 M, equation 1 M)

Q.5 Attempt any TWO of the following: (12 M)

a) Three moment theorem with fixed end — SFD and BMD

6 M



Step 1 — Free bending moments and $6A\bar{x}/L$ terms (each span treated as simply supported). For UDL: $6A\bar{x}/L = wL^3/4$; for point load W at distance a from the left end: $6A\bar{x}/L = Wab(L + a)/L$ about the left end and $Wab(L + b)/L$ about the right end.

Span	L (m)	Max free BM (kN·m)	Area A	$\bar{x}$ from left (m)	$\bar{x}$ from right (m)	$6A\bar{x}_{\text{left}}/L$	$6A\bar{x}_{\text{right}}/L$
AB	6	$Wab/L = 60 \times 2 \times 4/6 = 80$	240	2.67	3.33	640	800
BC	4	$wL^2/8 = 15 \times 4^2/8 = 30$	80	2	2	240	240

Step 2 — Clapeyron's equations (support moments M taken hogging positive):

$$M_1 L_1 + 2M_2(L_1 + L_2) + M_3 L_2 = 6A_1 \bar{x}_1/L_1 + 6A_2 \bar{x}_2/L_2$$

($\bar{x}_1$ measured from the left support of the left span, $\bar{x}_2$ from the right support of the right span.)

Since end A is fixed, an **imaginary span A_0A** of zero length ($L_0 = 0$, no load, $M_{A0} = 0$) is added to the left of A.

End C is simply supported, so $M_C = 0$.

For supports **A_0, A, B** : $12 M_A + 6 M_B = 800 \dots (1)$

For supports **A, B, C** : $6 M_A + 20 M_B = 880 \dots (2)$

Solving the simultaneous equations: $M_A = 52.55 \text{ kN}\cdot\text{m}$, $M_B = 28.24 \text{ kN}\cdot\text{m}$ (hogging).

Step 3 — Reactions, SFD and BMD

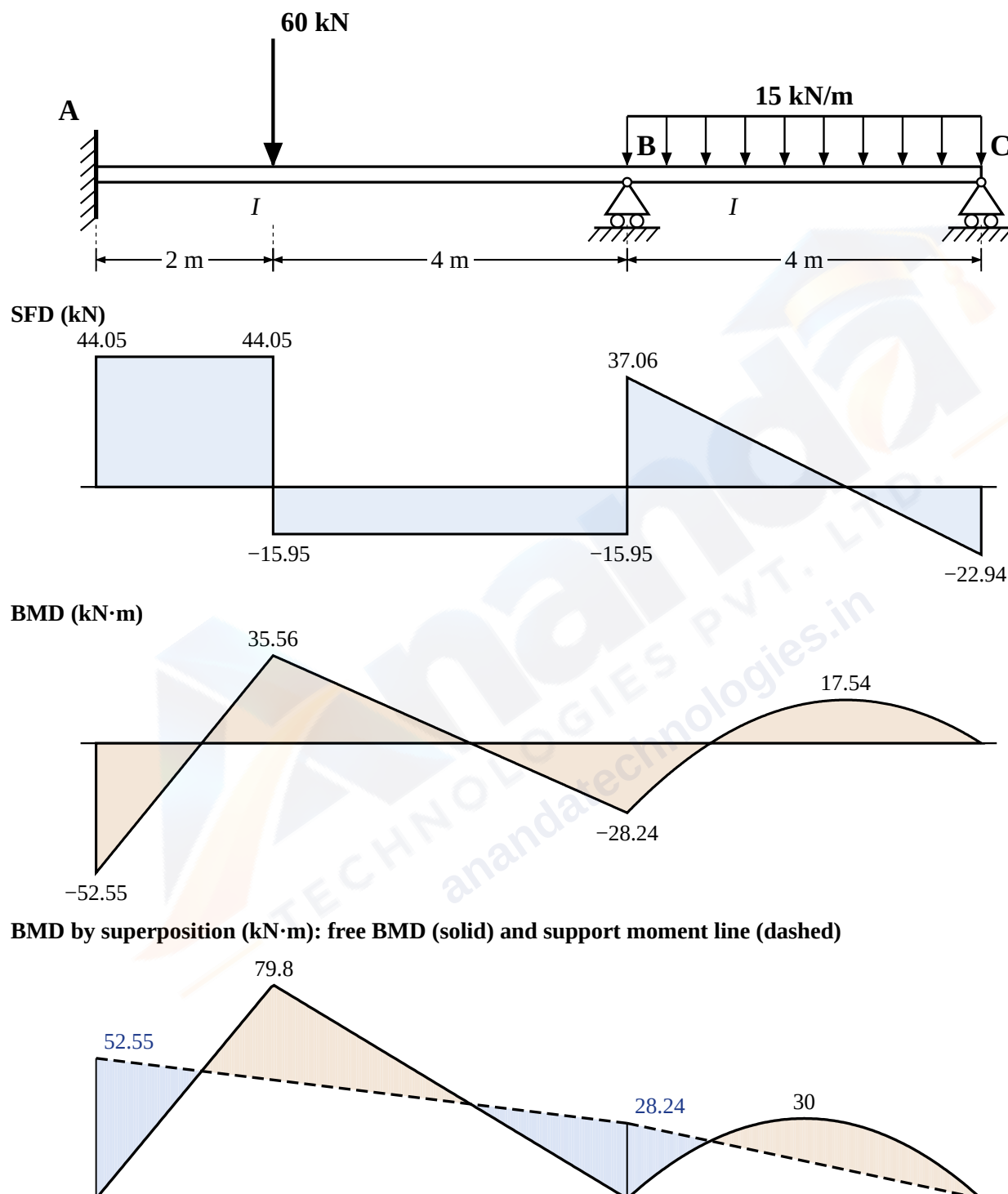
Support reactions (span-wise: free reaction $\pm (M_{\text{left}} - M_{\text{right}})/L$):

Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
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AB (6 m)	40 / 20	$(52.55 - 28.24)/6 = 4.05$	44.05	15.95
BC (4 m)	30 / 30	$(28.24 - 0)/4 = 7.06$	37.06	22.94

$R_A = 44.05 \text{ kN}$; $R_B = 53.01 \text{ kN}$; $R_C = 22.94 \text{ kN}$ ($\Sigma R = 120 \text{ kN} = \text{total load } \checkmark$)

Maximum sagging moments — span AB: **35.54 kN·m** at 2 m from A; span BC: **17.54 kN·m** at 2.47 m from B.

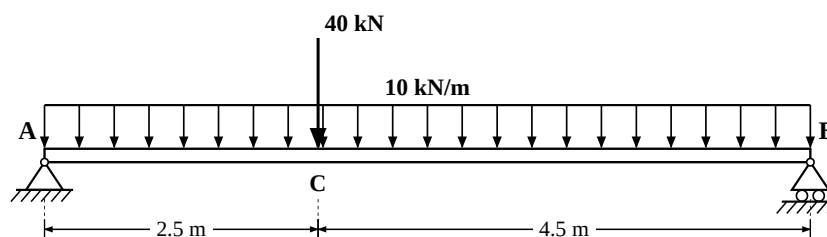


SFD, BMD (net) and BMD by superposition (+ sagging, - hogging)

✓ Check: support moments and reactions agree with an independent stiffness-matrix (and moment distribution) solution.
(Imaginary span and equations 2 M, support moments 2 M, SFD 1 M, BMD 1 M)

b) Macaulay's method — maximum deflection

6 M



Step 1 — Reactions: $\Sigma M_A = 0$: $R_B \times 7 = 10 \times 7 \times 3.5 + 40 \times 2.5 \rightarrow R_B = 49.29 \text{ kN}$; $R_A = (\text{total load}) - R_B = 60.71 \text{ kN}$

Step 2 — Macaulay's equation: Take A as origin and a section X-X at distance x from A in the last segment. (Terms after the vertical bar apply only when the bracket is positive.)

$$EI \frac{d^2y}{dx^2} = M_x = 60.714x - 5x^2 | - 40[x - 2.5]$$

$$EI \frac{dy}{dx} = 30.357x^2 - 1.667x^3 | - 20[x - 2.5]^2 + C_1 \quad \dots (1)$$

$$EI y = 10.119x^3 - 0.417x^4 | - 6.667[x - 2.5]^3 + C_1x + C_2 \quad \dots (2)$$

Step 3 — Boundary conditions: At $x = 0$, $y = 0 \rightarrow$ (first segment of eq. 2) $C_2 = 0$. At $x = 7 \text{ m}$, $y = 0 \rightarrow$ (all terms of eq. 2) $C_1 = -266.131$

Step 4 — Results:

- Maximum deflection:** it occurs where slope = 0. Slope at $x = 2.5 \text{ m}$ is negative and at $x = 7 \text{ m}$ is positive, so the point lies between $x = 2.5 \text{ m}$ and 7 m . Using the terms of eq. (1) valid in this segment:

$$EI \frac{dy}{dx} = -1.667x^3 + 10.357x^2 + 100x - 391.131 = 0 \rightarrow \text{solving (trial / calculator), } x = 3.373 \text{ m from A}$$

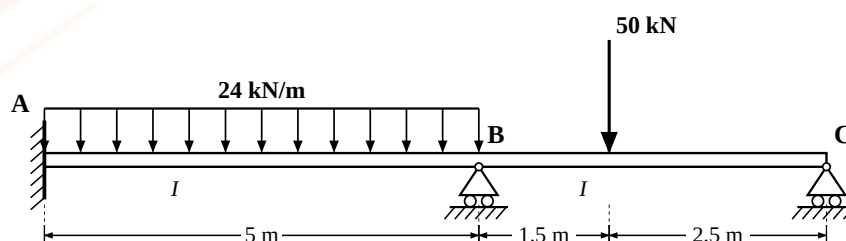
$$\text{Putting } x = 3.373 \text{ m in eq. (2): } EI y_{\max} = -567.71 \rightarrow y_{\max} = 567.71/EI = 567.71/30000 \text{ m} = 18.92 \text{ mm (downward)}$$

✓ Check: values agree with an independent stiffness-matrix analysis of the same beam.

(Reactions 1 M, Macaulay equation 1 M, constants 1 M, location 2 M, value 1 M)

c) Moment distribution — two-span beam

6 M



Step 1 — Fixed end moments (clockwise +, anticlockwise -):

Span	Left end formula	$M_{\text{left}} \text{ (kN}\cdot\text{m)}$	Right end formula	$M_{\text{right}} \text{ (kN}\cdot\text{m)}$
AB	$wL^2/12 = 24 \times 5^2/12$	-50	$wL^2/12$	+50
BC	$Wab^2/L^2 = 50 \times 1.5 \times 2.5^2/4^2$	-29.3	$Wa^2b/L^2 = 50 \times 1.5^2 \times 2.5/4^2$	+17.58

Step 2 — Stiffness and distribution factors ($k = I/L$ for far end fixed or continuous; $k = (3/4) \cdot I/L$ when the far end is simply supported):

Joint	Member	Relative stiffness k	Σk	Distribution factor
A	AB	$I/5 = 0.2I$	$0.2I$	0 (fixed end)
B	BA	$I/5 = 0.2I$	$0.3875I$	$0.2/0.3875 = \mathbf{0.516}$
	BC	$(3/4) \cdot I/4 = 0.1875I$		$0.1875/0.3875 = \mathbf{0.484}$
C	CB	$I/4 = 0.25I$	$0.25I$	1 (simple end)

Step 3 — Moment distribution table (carry-over factor $\frac{1}{2}$; no carry-over to a simply supported end; the simple end is first released and half of the released moment is carried over):

Step	A	B		C
	AB	BA	BC	CB
D.F.	0	0.516	0.484	1
FEM	-50	50	-29.3	17.58
Release simple end				-17.58
Carry over			-8.79	
Balance		-6.15	-5.76	
Carry over	-3.07			
Final moments	-53.07	43.85	-43.85	0

Support moments: $M_A = 53.07 \text{ kN}\cdot\text{m}$; $M_B = 43.85 \text{ kN}\cdot\text{m}$; $M_C = 0 \text{ kN}\cdot\text{m}$ (hogging; final end moments at a joint are equal and opposite ✓).

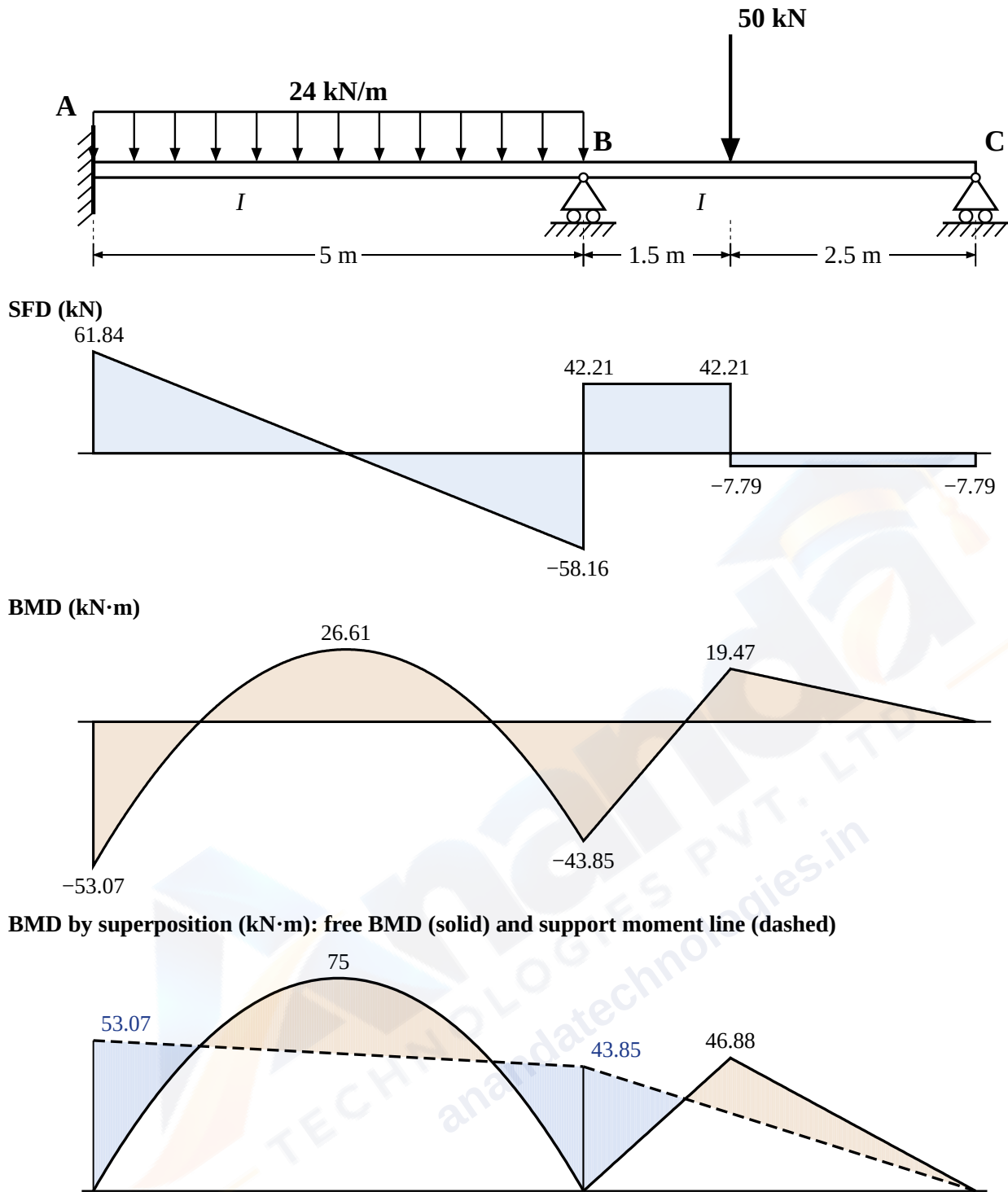
Step 4 — Reactions and diagrams

Support reactions (span-wise: free reaction $\pm (M_{\text{left}} - M_{\text{right}})/L$):

Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (5 m)	60 / 60	$(53.07 - 43.85)/5 = 1.84$	61.84	58.16
BC (4 m)	31.25 / 18.75	$(43.85 - 0)/4 = 10.96$	42.21	7.79

$R_A = 61.84 \text{ kN}$; $R_B = 100.37 \text{ kN}$; $R_C = 7.79 \text{ kN}$ ($\Sigma R = 170 \text{ kN} = \text{total load}$ ✓)

Maximum sagging moments — span AB: **$26.61 \text{ kN}\cdot\text{m}$** at 2.58 m from A; span BC: **$19.47 \text{ kN}\cdot\text{m}$** at 1.5 m from B.



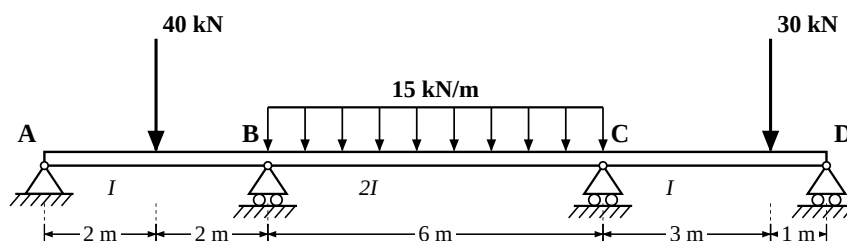
SFD, BMD (net) and BMD by superposition (+ sagging, - hogging)

✓ Check: support moments agree with the theorem of three moments and with a stiffness-matrix solve (difference < 0.01 kN·m).
(FEM 1 M, DF 1 M, distribution 2 M, SFD and BMD 2 M)

Q.6 Attempt any TWO of the following: (12 M)

a) Moment distribution — three-span beam

6 M



Step 1 — Fixed end moments (clockwise +, anticlockwise -):

Span	Left end formula	$M_{\text{left}} \text{ (kN}\cdot\text{m)}$	Right end formula	$M_{\text{right}} \text{ (kN}\cdot\text{m)}$
AB	$WL/8 = 40 \times 4/8$	-20	$WL/8$	+20
BC	$wL^2/12 = 15 \times 6^2/12$	-45	$wL^2/12$	+45
CD	$Wab^2/L^2 = 30 \times 3 \times 1^2/4^2$	-5.62	$Wa^2b/L^2 = 30 \times 3^2 \times 1/4^2$	+16.88

Step 2 — Stiffness and distribution factors ($k = I/L$ for far end fixed or continuous; $k = (3/4) \cdot I/L$ when the far end is simply supported):

Joint	Member	Relative stiffness k	Σk	Distribution factor
A	AB	$I/4 = 0.25I$	0.25I	1 (simple end)
B	BA	$(3/4) \cdot I/4 = 0.1875I$	0.5208I	$0.1875/0.5208 = 0.36$
	BC	$2I/6 = 0.3333I$		$0.3333/0.5208 = 0.64$
C	CB	$2I/6 = 0.3333I$	0.5208I	$0.3333/0.5208 = 0.64$
	CD	$(3/4) \cdot I/4 = 0.1875I$		$0.1875/0.5208 = 0.36$
D	DC	$I/4 = 0.25I$	0.25I	1 (simple end)

Step 3 — Moment distribution table (carry-over factor $1/2$; no carry-over to a simply supported end; the simple end is first released and half of the released moment is carried over):

Step	A	B		C		D
	AB	BA	BC	CB	CD	DC
D.F.	1	0.36	0.64	0.64	0.36	1
FEM	-20	20	-45	45	-5.62	16.88
Release simple end	20					-16.88
Carry over		10			-8.44	
Balance		5.4	9.6	-19.8	-11.14	
Carry over			-9.9	4.8		
Balance		3.56	6.34	-3.07	-1.73	
Carry over			-1.54	3.17		
Balance		0.55	0.98	-2.03	-1.14	
Carry over			-1.01	0.49		

Balance		0.36	0.65	-0.31	-0.18	
Carry over			-0.16	0.32		
Balance		0.06	0.1	-0.21	-0.12	
Carry over			-0.1	0.05		
Balance		0.04	0.07	-0.03	-0.02	
Carry over			-0.02	0.03		
Balance		0.01	0.01	-0.02	-0.01	
Carry over			-0.01	0.01		
Final moments	0	39.98	-39.99	28.4	-28.39	0

Support moments: $M_A = 0 \text{ kN}\cdot\text{m}$; $M_B = 39.98 \text{ kN}\cdot\text{m}$; $M_C = 28.4 \text{ kN}\cdot\text{m}$; $M_D = 0 \text{ kN}\cdot\text{m}$ (hogging; final end moments at a joint are equal and opposite ✓).

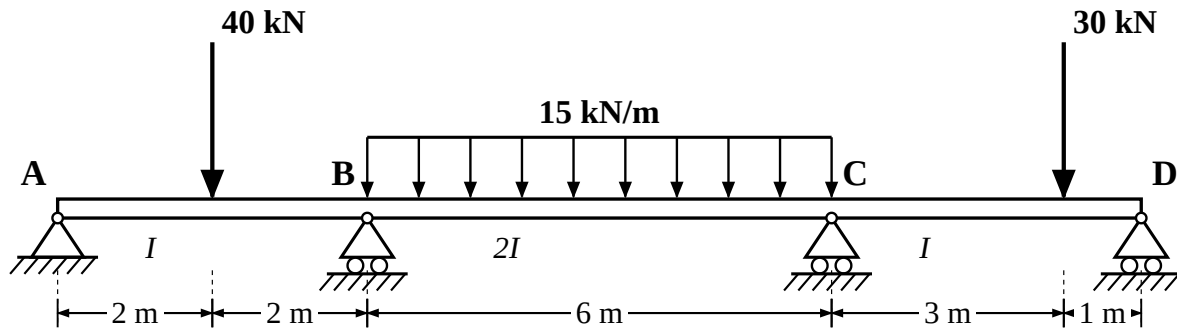
Step 4 — Reactions and diagrams

Support reactions (span-wise: free reaction $\pm (M_{\text{left}} - M_{\text{right}})/L$):

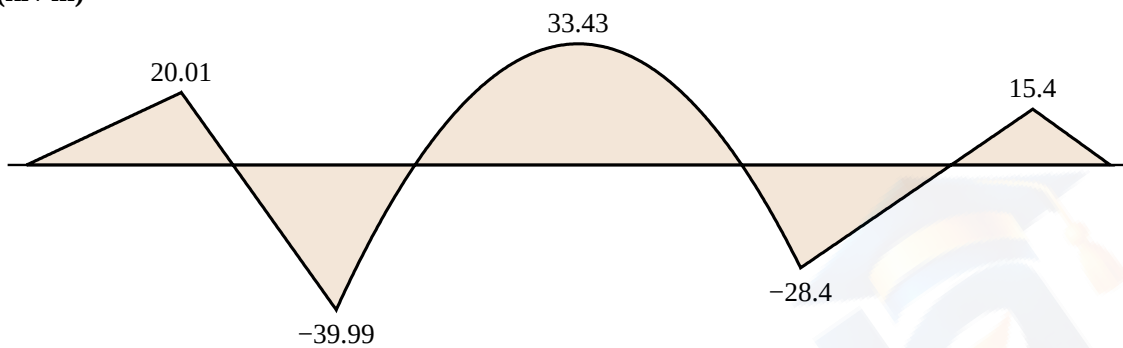
Span	Free reactions (left/right) kN	Correction (kN)	Left end (kN)	Right end (kN)
AB (4 m)	20 / 20	$(0 - 39.98)/4 = -10$	10	30
BC (6 m)	45 / 45	$(39.98 - 28.4)/6 = 1.93$	46.93	43.07
CD (4 m)	7.5 / 22.5	$(28.4 - 0)/4 = 7.1$	14.6	15.4

$R_A = 10 \text{ kN}$; $R_B = 76.93 \text{ kN}$; $R_C = 57.67 \text{ kN}$; $R_D = 15.4 \text{ kN}$ ($\Sigma R = 160 \text{ kN} = \text{total load}$ ✓)

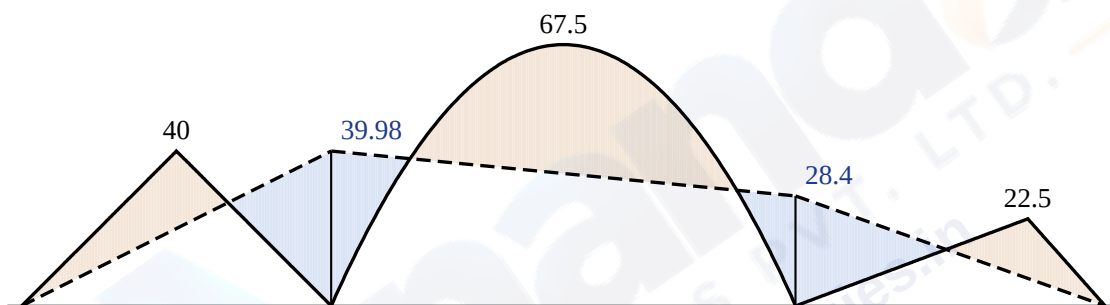
Maximum sagging moments — span AB: **$20.01 \text{ kN}\cdot\text{m}$** at 2 m from A; span BC: **$33.43 \text{ kN}\cdot\text{m}$** at 3.13 m from B; span CD: **$15.4 \text{ kN}\cdot\text{m}$** at 3 m from C.



BMD (kN·m)



BMD by superposition (kN·m): free BMD (solid) and support moment line (dashed)



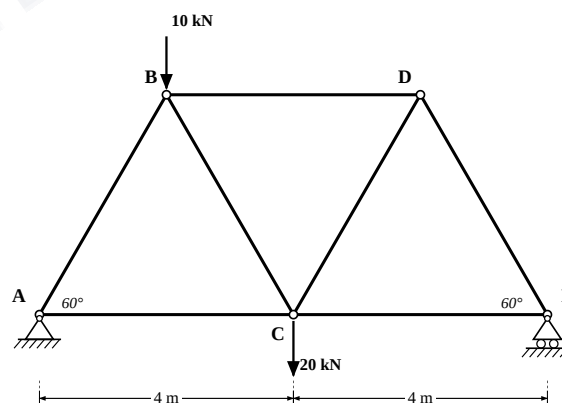
BMD (net) and BMD by superposition (+ sagging, - hogging)

✓ Check: support moments agree with the theorem of three moments and with a stiffness-matrix solve (difference < 0.01 kN·m).

(FEM 1 M, DF 1 M, distribution table 2 M, support moments 1 M, BMD 1 M)

b) Truss — method of joints

6 M



Check: number of joints $j = 5$, members $m = 7$; $2j - 3 = 7 = m \rightarrow$ **perfect frame.**

Support reactions: $\Sigma M_A = 0$: $R_E \times 8 = 10 \times 2 + 20 \times 4 \rightarrow R_E = 12.5 \text{ kN } (\uparrow)$; $\Sigma F_y = 0$: $R_A = 30 - 12.5 = 17.5 \text{ kN } (\uparrow)$; $\Sigma F_x = 0$: $H_A = 0$ (all loads vertical).

All unknown member forces are first assumed **tensile** (acting away from the joint). A negative answer means the member is in **compression**.

Joint A (unknowns: F_{AB} , F_{AC}):

$$\Sigma F_x = 0: F_{AB} \cos 60^\circ + F_{AC} = 0$$

$$\Sigma F_y = 0: F_{AB} \sin 60^\circ + 17.5 = 0$$

$$\rightarrow F_{AB} = -20.21 \text{ kN (C)}, F_{AC} = 10.1 \text{ kN (T)}$$

Joint B (unknowns: F_{BC} , F_{BD}):

$$\Sigma F_x = 0: -(-20.21) \cos 60^\circ + F_{BC} \cos 60^\circ + F_{BD} = 0$$

$$\Sigma F_y = 0: -(-20.21) \sin 60^\circ - F_{BC} \sin 60^\circ - 10 = 0$$

$$\rightarrow F_{BC} = 8.66 \text{ kN (T)}, F_{BD} = -14.43 \text{ kN (C)}$$

Joint C (unknowns: F_{CD} , F_{CE}):

$$\Sigma F_x = 0: -(8.66) \cos 60^\circ + F_{CD} \cos 60^\circ - (10.1) + F_{CE} = 0$$

$$\Sigma F_y = 0: (8.66) \sin 60^\circ + F_{CD} \sin 60^\circ - 20 = 0$$

$$\rightarrow F_{CD} = 14.43 \text{ kN (T)}, F_{CE} = 7.22 \text{ kN (T)}$$

Joint D (unknowns: F_{DE}):

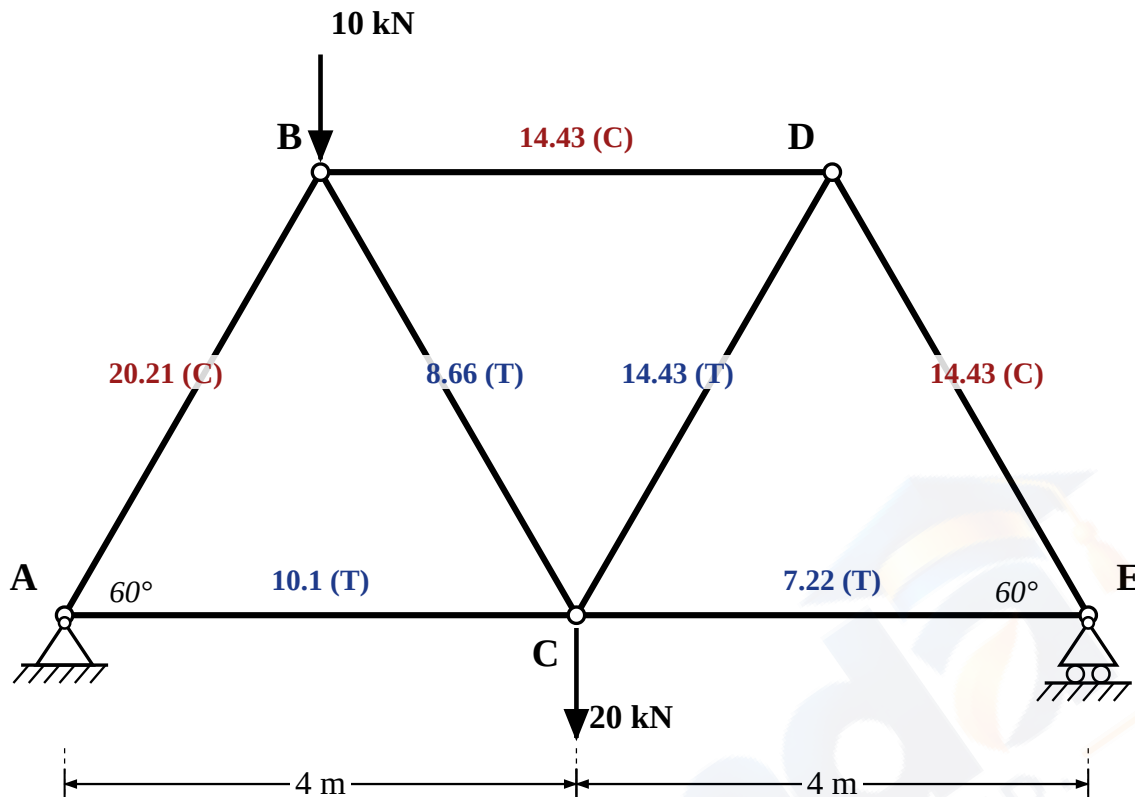
$$\Sigma F_x = 0: -(14.43) \cos 60^\circ + F_{DE} \cos 60^\circ - (-14.43) = 0$$

$$\Sigma F_y = 0: -(14.43) \sin 60^\circ - F_{DE} \sin 60^\circ = 0$$

$$\rightarrow F_{DE} = -14.43 \text{ kN (C)}$$

Result table

Member	Force (kN)	Nature
AB	20.21	Compression
BC	8.66	Tension
CD	14.43	Tension
DE	14.43	Compression
BD	14.43	Compression
AC	10.1	Tension
CE	7.22	Tension



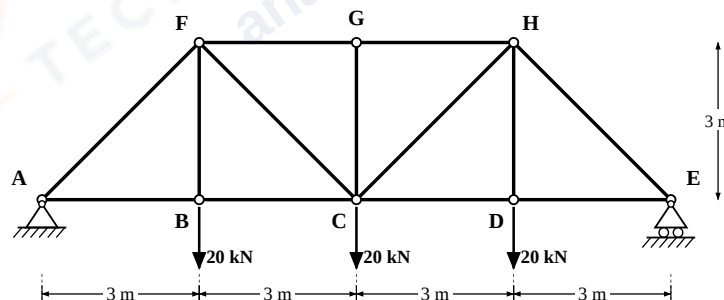
Member forces (T = tension, C = compression)

✓ Check: all member forces agree with a full matrix solution of the 2j joint-equilibrium equations.

(Reactions 1 M, joint equations 3 M, results with nature 2 M)

c) Truss — method of sections

6 M



Support reactions: $\Sigma M_A = 0$: $R_E \times 12 = 20 \times 3 + 20 \times 6 + 20 \times 9 \rightarrow R_E = 30 \text{ kN } (\uparrow)$; $\Sigma F_y = 0$: $R_A = 60 - 30 = 30 \text{ kN } (\uparrow)$; $\Sigma F_x = 0$: $H_A = 0$ (all loads vertical).

Draw section line 1–1 cutting members FG, FC, BC and consider the equilibrium of the part containing joints A, B, F. Forces in cut members are assumed tensile (away from the part).

Force in FG: take moments about C (anticlockwise +). Perpendicular distance of FG from this point = 3 m.

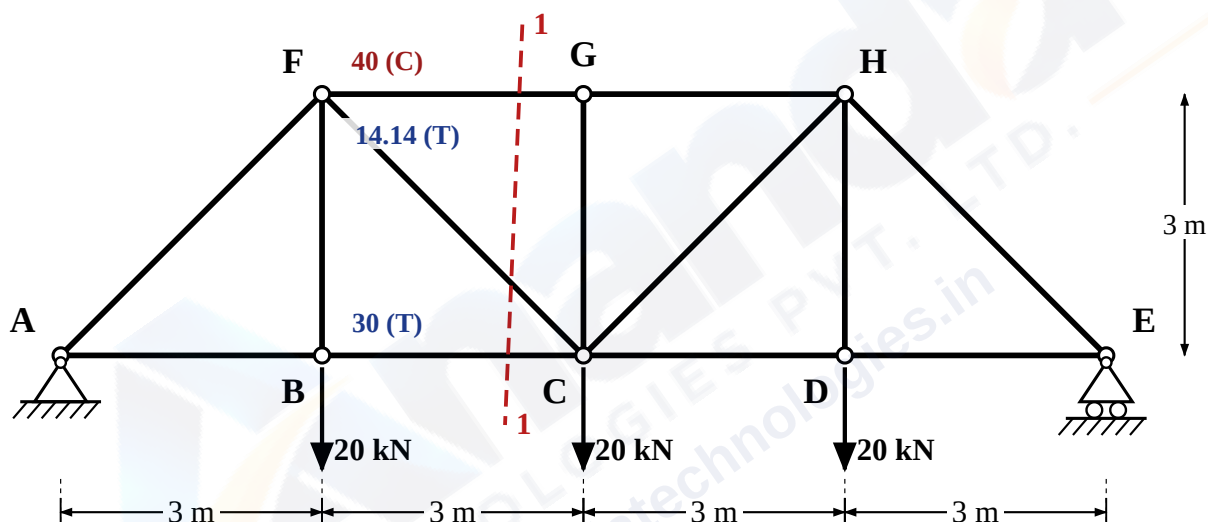
$$\Sigma M = 0: 20 \times 3 - 30 \times 6 - F_{FG} \times 3 = 0 \rightarrow F_{FG} = -40 \text{ kN (C)}$$

Force in FC: members FG and BC are horizontal (parallel), so resolve vertically: $\Sigma F_y = 0: -20 + 30 - F_{FC} \sin 45^\circ = 0 \rightarrow F_{FC} = 14.14 \text{ kN (T)}$

Force in BC: take moments about F (anticlockwise +). Perpendicular distance of BC from this point = 3 m.

$$\Sigma M = 0: -30 \times 3 + F_{BC} \times 3 = 0 \rightarrow F_{BC} = 30 \text{ kN (T)}$$

Member	Force (kN)	Nature
FG	40	Compression
FC	14.14	Tension
BC	30	Tension



Section 1-1 and forces in the cut members

✓ Check: values agree with the full matrix (method of joints) solution of the truss.

(Reactions 1 M, section 1 M, each force with nature 4 M)

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