

Strength of Materials (313308)

Model Question Paper — Set A · Model Answers

Units: N, mm, N/mm² (= MPa) unless stated. 1 kN = 10³ N, 1 kN·m = 10⁶ N·mm, 1 GPa = 10³ N/mm².

Q.1

a) Moment of inertia and radius of gyration.

2 M

Moment of inertia (second moment of area): The moment of inertia of a plane area about an axis is the sum of the products of each elemental area and the square of its distance from that axis. $I = \sum a \cdot y^2$ (unit: mm⁴ or m⁴).

Radius of gyration (k): It is the distance from the axis at which the whole area may be assumed to be concentrated so that it gives the same moment of inertia. $I = A \cdot k^2 \Rightarrow k = \sqrt{(I/A)}$ (unit: mm).

(Each definition 1 M)

b) Hooke's law and Poisson's ratio.

2 M

Hooke's law: Within the elastic limit, stress is directly proportional to strain. $\sigma \propto e \Rightarrow \sigma = E \cdot e$, where E is the modulus of elasticity.

Poisson's ratio (μ or $1/m$): Within the elastic limit, the ratio of lateral strain to the corresponding linear (longitudinal) strain is constant for a material. $\mu = \text{lateral strain} / \text{linear strain}$ (value 0.25 to 0.33 for metals).

(1 M each)

c) Point of contraflexure and point of contra-shear.

2 M

Point of contraflexure: The point on a beam where the bending moment changes its sign (from sagging to hogging or vice versa); B.M. is zero at this point. It occurs in overhanging beams.

Point of contra-shear: The point on a beam where the shear force changes its sign (S.F. = 0). The bending moment is maximum at this point.

(1 M each)

d) Assumptions in theory of pure bending.

2 M

1. The material of the beam is homogeneous and isotropic.
2. The beam material obeys Hooke's law; stresses are within the elastic limit.
3. The value of E is the same in tension and compression.
4. Transverse sections which are plane before bending remain plane after bending.

(Any four, ½ M each)

e) Single shear vs double shear.

2 M

Single shear	Double shear
The member (rivet / pin) is likely to be sheared at one cross-section.	The member is likely to be sheared at two cross-sections.

Resisting area = $\pi d^2/4$.	Resisting area = $2 \times \pi d^2/4$.
Shear stress $\tau = P / (\pi d^2/4)$.	Shear stress $\tau = P / (2 \times \pi d^2/4)$ — half that of single shear for the same load.
Example: rivet in a lap joint.	Example: rivet in a butt joint with two cover plates, pin of a knuckle joint.

(Any two points, 1 M each)

f) Direct load and eccentric load.

2 M

Direct (axial) load: A load whose line of action coincides with the longitudinal (centroidal) axis of the member. It produces only uniform direct stress $\sigma_0 = P/A$ over the section.

Eccentric load: A load whose line of action is parallel to the axis of the member but at some distance e (eccentricity) from it. It produces direct stress P/A plus bending stress due to moment $M = P \cdot e$.

(1 M each)

g) Maximum and average shear stress relation.

2 M

Average shear stress $\tau_{avg} = F/A$.

- **Rectangular section:** $\tau_{max} = 1.5 \tau_{avg}$ (at the neutral axis).
- **Circular section:** $\tau_{max} = (4/3) \tau_{avg}$ (at the neutral axis).

(1 M each)

Q.2

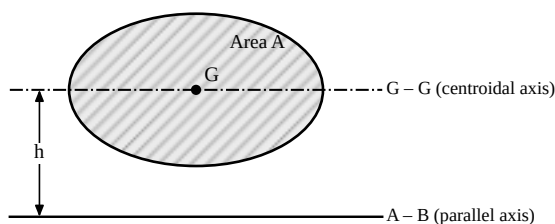
a) Parallel axis and perpendicular axis theorems.

4 M

Parallel axis theorem: The moment of inertia of a plane area about any axis parallel to its centroidal axis is equal to the moment of inertia about the centroidal axis plus the product of the area and the square of the distance between the two axes.

$$I_{AB} = I_G + A \cdot h^2$$

where I_G = M.I. about centroidal axis, A = area of lamina, h = distance between the two parallel axes.

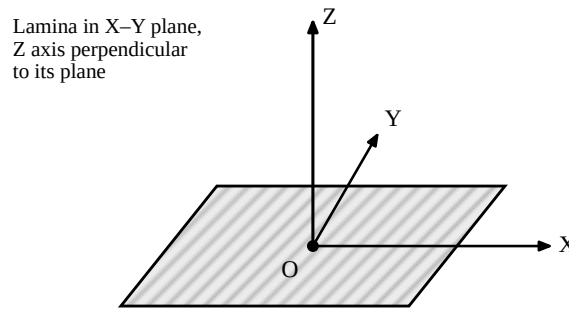


$$\text{Parallel axis theorem: } I_{AB} = I_G + A \cdot h^2$$

Perpendicular axis theorem: The moment of inertia of a plane lamina about an axis perpendicular to its plane (Z-Z) and passing through a point O is equal to the sum of its moments of inertia about two mutually perpendicular axes X-X and Y-Y lying in the plane of the lamina and intersecting at O.

$$I_{ZZ} = I_{XX} + I_{YY}$$

I_{ZZ} is also called the polar moment of inertia (J).

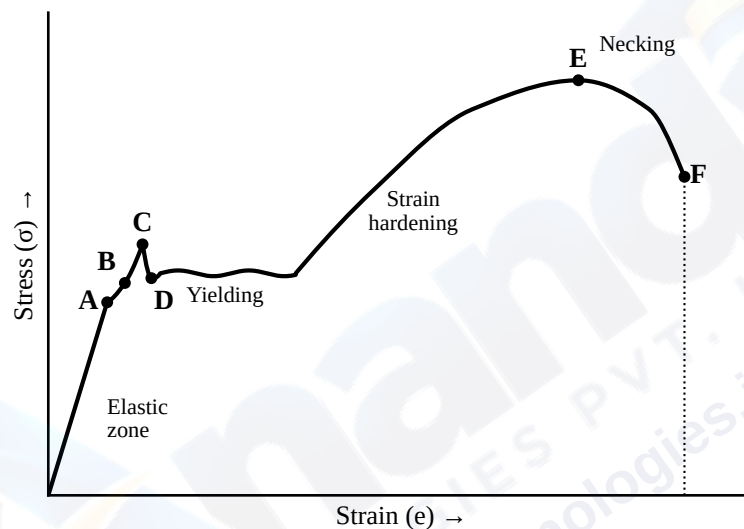


Perpendicular axis theorem: $I_{ZZ} = I_{XX} + I_{YY}$

(Each theorem with formula 1 M, sketch 1 M)

b) Stress-strain diagram for mild steel.

4 M



A – Limit of proportionality, B – Elastic limit, C – Upper yield point,
D – Lower yield point, E – Ultimate stress point, F – Breaking point

Stress-strain curve for mild steel (ductile material) in tension

- **A – Limit of proportionality:** Up to A, stress is directly proportional to strain (Hooke's law); OA is a straight line.
- **B – Elastic limit:** Maximum stress up to which the material regains its original shape fully on removal of load.
- **C – Upper yield point:** Stress at which the material starts yielding; strain increases suddenly without increase in load.
- **D – Lower yield point:** Stress drops slightly and remains nearly constant while large plastic strain occurs (yielding).
- **E – Ultimate stress point:** Maximum stress the material can take = maximum load / original area. After E, necking starts.
- **F – Breaking point:** The specimen breaks; stress at this point is the breaking stress.

(Diagram 2 M, explanation 2 M)

c) Stress, strain, elongation and change in diameter of steel rod.

4 M

Given: $d = 25 \text{ mm}$, $L = 2000 \text{ mm}$, $P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$, $E = 200 \text{ GPa} = 2 \times 10^5 \text{ N/mm}^2$, $\mu = 0.3$

$$\text{Area } A = \pi d^2/4 = \pi \times 25^2/4 = 490.87 \text{ mm}^2$$

$$\text{Stress } \sigma = P/A = 60 \times 10^3 / 490.87 = \mathbf{122.231 \text{ N/mm}^2}$$

$$\text{Linear strain } e = \sigma/E = 122.231 / (2 \times 10^5) = \mathbf{6.1115 \times 10^{-4}}$$

$$\text{Elongation } \delta L = e \times L = 6.1115 \times 10^{-4} \times 2000 = \mathbf{1.2223 \text{ mm}}$$

$$\text{Lateral strain} = \mu \times e = 0.3 \times 6.1115 \times 10^{-4} = 1.8335 \times 10^{-4}$$

$$\text{Change in diameter } \delta d = \text{lateral strain} \times d = 1.8335 \times 10^{-4} \times 25 = \mathbf{0.00458 \text{ mm (decrease)}}$$

(Stress 1 M, strain 1 M, δL 1 M, δd 1 M)

d) Cantilever 4 m — SFD and BMD.

4 M

Reaction at fixed end A: $R_A = \text{total load} = 8 + 2 \times 4 = \mathbf{16 \text{ kN} (\uparrow)}$

Fixing moment at A = $40 \text{ kN}\cdot\text{m}$ (hogging). For a cantilever, S.F. and B.M. are calculated from the free end.

Shear force calculations (from the free end (right side), upward force on left = +ve):

$$\text{S.F. just right of A} = 8 + 2 \times 4 = \mathbf{16 \text{ kN}}$$

$$\text{S.F. just left of B} = 8 + 2 \times 1 = \mathbf{10 \text{ kN}}$$

$$\text{S.F. just right of B} = 2 \times 1 = \mathbf{2 \text{ kN}}$$

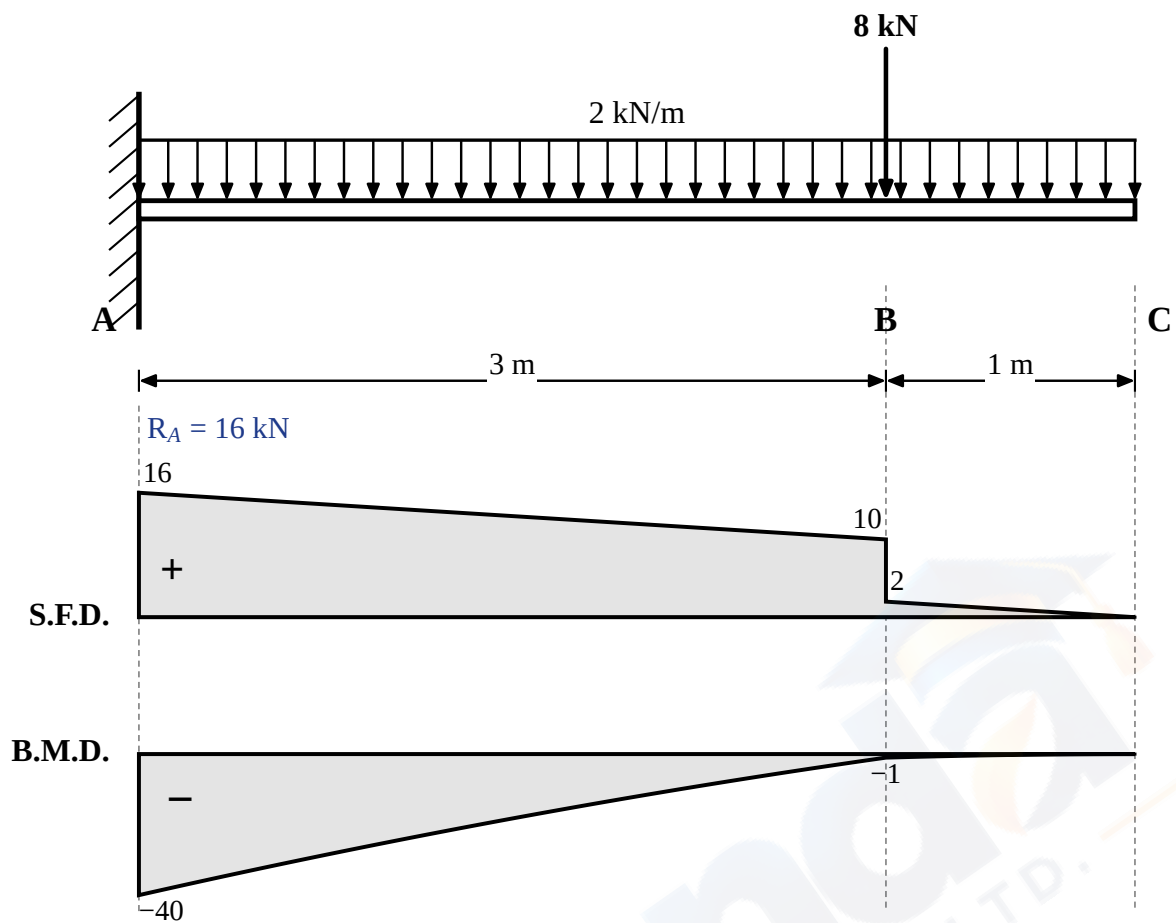
$$\text{S.F. just left of C} = \mathbf{0 \text{ kN}}$$

Bending moment calculations (sagging = +ve, hogging = -ve):

$$M_A = -8 \times 3 - 2 \times 4 \times 2 = \mathbf{-40 \text{ kN}\cdot\text{m}}$$

$$M_B = -2 \times 1 \times 0.5 = \mathbf{-1 \text{ kN}\cdot\text{m}}$$

$$M_C = 0 \text{ (free end)}$$

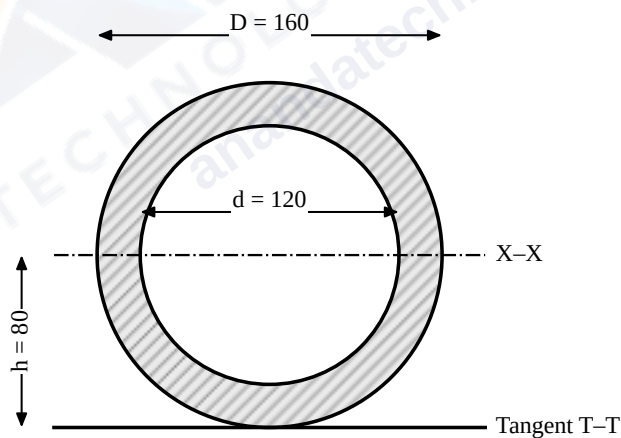


S.F. and B.M. diagrams

Q.3

a) Hollow circular section — I about centroid, tangent and polar MI.

4 M



Hollow circular section with tangent

$$I_{XX} = \pi(D^4 - d^4)/64 = \pi(160^4 - 120^4)/64 = 2.1991 \times 10^7 \text{ mm}^4$$

$$\text{Area } A = \pi(D^2 - d^2)/4 = \pi(160^2 - 120^2)/4 = 8796.46 \text{ mm}^2$$

About tangent (parallel axis theorem, $h = D/2 = 80$ mm):

$$I_T = I_{XX} + A \cdot h^2 = 2.1991 \times 10^7 + 8796.46 \times 80^2 = 7.8288 \times 10^7 \text{ mm}^4$$

Polar M.I. (perpendicular axis theorem): $J = I_{XX} + I_{YY} = 2 \times 2.1991 \times 10^7 = 4.3982 \times 10^7 \text{ mm}^4$

(I centroidal 1 M, I tangent 2 M, J 1 M)

b) Poisson's ratio and bulk modulus.

4 M

Relations: $E = 2G(1 + \mu)$ and $E = 3K(1 - 2\mu)$

From $E = 2G(1 + \mu)$: $2.1 \times 10^5 = 2 \times 0.8 \times 10^5 (1 + \mu) \Rightarrow 1 + \mu = 2.1/1.6 = 1.3125 \Rightarrow \mu = 0.3125$

From $E = 3K(1 - 2\mu)$: $K = E / [3(1 - 2\mu)] = 2.1 \times 10^5 / [3 \times (1 - 2 \times 0.3125)] = 2.1 \times 10^5 / 1.125 = 1.8667 \times 10^5 \text{ N/mm}^2$

(μ 2 M, K 2 M)

c) Simply supported beam 6 m — SFD, BMD and maximum BM.

4 M

Support reactions (R_A and R_D act upward):

Taking moments about A, $\Sigma M_A = 0$:

$$R_D \times 6 = (3 \times 3) \times 1.5 + 8 \times 4.5 = 49.5$$

$$\therefore R_D = 49.5 / 6 = 8.25 \text{ kN}$$

$$\Sigma F_y = 0: R_A + R_D = 8 + 3 \times 3 = 17 \text{ kN}$$

$$\therefore R_A = 17 - 8.25 = 8.75 \text{ kN}$$

Shear force calculations (from the left, upward force on left = +ve):

$$\text{S.F. just right of A} = 8.75 = 8.75 \text{ kN}$$

$$\text{S.F. at B} = 8.75 - 3 \times 3 = -0.25 \text{ kN}$$

$$\text{S.F. just left of C} = 8.75 - 3 \times 3 = -0.25 \text{ kN}$$

$$\text{S.F. just right of C} = 8.75 - 3 \times 3 - 8 = -8.25 \text{ kN}$$

$$\text{S.F. just left of D} = 8.75 - 3 \times 3 - 8 = -8.25 \text{ kN}$$

Bending moment calculations (sagging = +ve, hogging = -ve):

$$M_A = 0 \text{ (simple support)}$$

$$M_B = 8.75 \times 3 - 3 \times 3 \times 1.5 = 12.75 \text{ kN}\cdot\text{m}$$

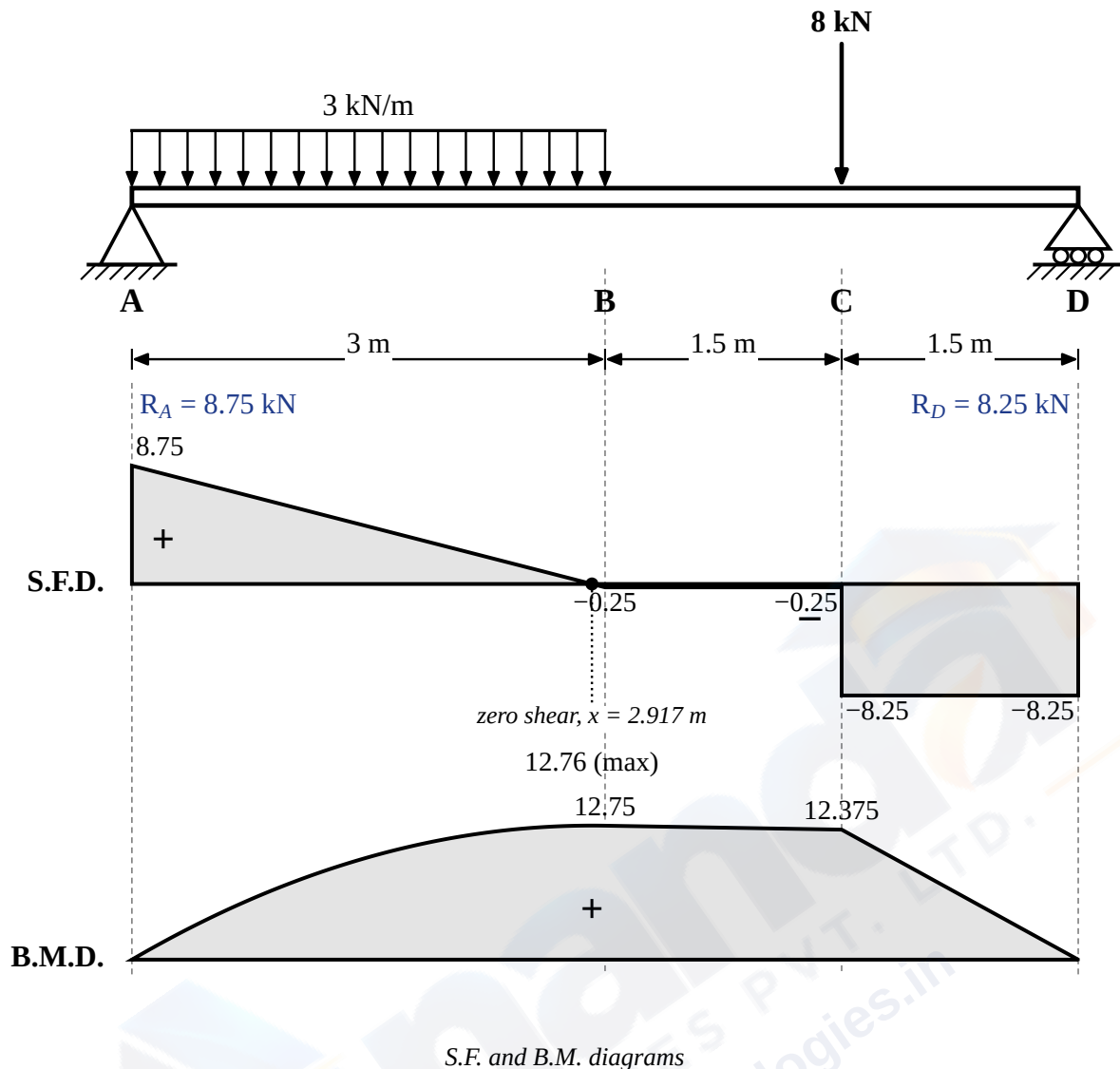
$$M_C = 8.75 \times 4.5 - 3 \times 3 \times 3 = 12.375 \text{ kN}\cdot\text{m}$$

$$M_D = 0 \text{ (simple support)}$$

Point of zero shear (contra-shear): let it be at x m from A. Between A and the next point S.F. =

$$8.75 - 3(x - 0) = 0 \Rightarrow x = 2.917 \text{ m from A.}$$

$$\text{Maximum B.M. at this section: } M_{\max} = 8.75 \times 2.917 - 3 \times 2.917 \times 1.458 = 12.76 \text{ kN}\cdot\text{m}$$



d) Rectangular column 300×200 — eccentric load.

4 M

Given: $P = 240$ kN, $e = 40$ mm along the 300 mm side ($b = 300$ mm, $d = 200$ mm).

$$\text{Area } A = 300 \times 200 = 60000 \text{ mm}^2$$

$$\text{Section modulus (about the axis } \perp \text{ to plane of eccentricity) } Z = \frac{d \cdot b^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 \text{ mm}^3$$

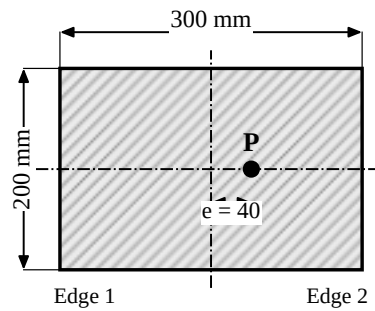
$$\text{Direct stress } \sigma_0 = \frac{P}{A} = \frac{2.4 \times 10^5}{60000} = 4 \text{ N/mm}^2 \text{ (C)}$$

$$\text{Bending moment } M = P \cdot e = 2.4 \times 10^5 \times 40 = 9.6 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\text{Bending stress } \sigma_b = \frac{M}{Z} = \frac{9.6 \times 10^6}{3 \times 10^6} = 3.2 \text{ N/mm}^2$$

$$\sigma_{\max} = \sigma_0 + \sigma_b = 4 + 3.2 = 7.2 \text{ N/mm}^2 \text{ (compressive)}$$

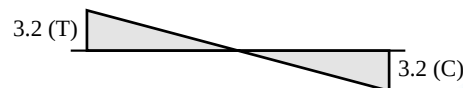
$$\sigma_{\min} = \sigma_0 - \sigma_b = 4 - 3.2 = 0.8 \text{ N/mm}^2 \text{ (compressive)}$$



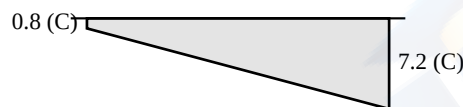
Direct stress σ_0



Bending stress σ_b



Resultant stress



(stresses in N/mm^2 ; C = compression, drawn below the line; T = tension, drawn above)

Fig.: Section with load position and stress distribution

(σ_0 1 M, σ_b 1 M, $\sigma_{\max}/\sigma_{\min}$ 1 M, diagram 1 M)

Q.4

a) Bending stress in SS beam of rectangular section.

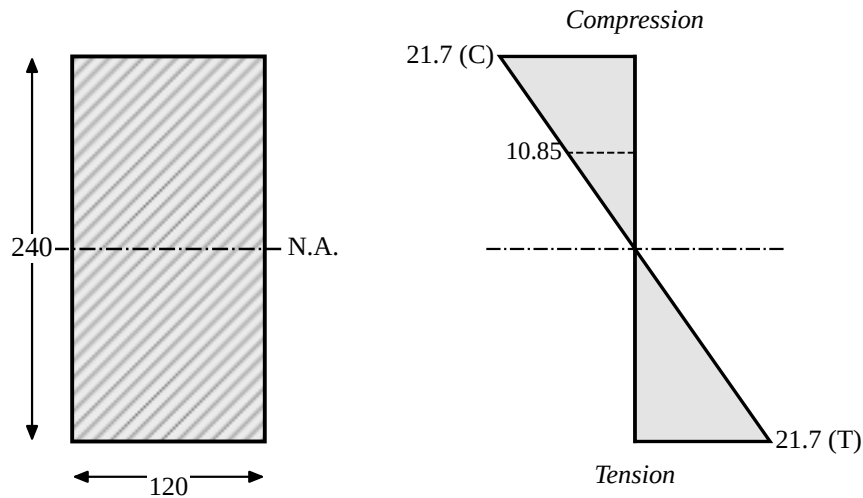
4 M

Maximum B.M. for full-span u.d.l.: $M = wL^2/8 = 8 \times 5^2/8 = 25 \text{ kN}\cdot\text{m} = 2.5 \times 10^7 \text{ N}\cdot\text{mm}$

$I = bd^3/12 = 120 \times 240^3/12 = 1.3824 \times 10^8 \text{ mm}^4$; $y_{\max} = 120 \text{ mm}$; $Z = bd^2/6 = 120 \times 240^2/6 = 1.152 \times 10^6 \text{ mm}^3$

Flexure formula $M/I = \sigma/y \Rightarrow \sigma_{\max} = M/Z = 2.5 \times 10^7 / 1.152 \times 10^6 = 21.701 \text{ N/mm}^2$ (compression at top, tension at bottom fibre)

At 60 mm from top, $y = 120 - 60 = 60 \text{ mm}$ from N.A.: $\sigma = M \cdot y/I = 2.5 \times 10^7 \times 60 / 1.3824 \times 10^8 = 10.851 \text{ N/mm}^2$ (compressive)



Bending stress distribution (stresses in N/mm²)

(M 1 M, σ_{max} 1 M, σ at 60 mm 1 M, diagram 1 M)

b) Shear stress distribution in rectangular beam section.

4 M

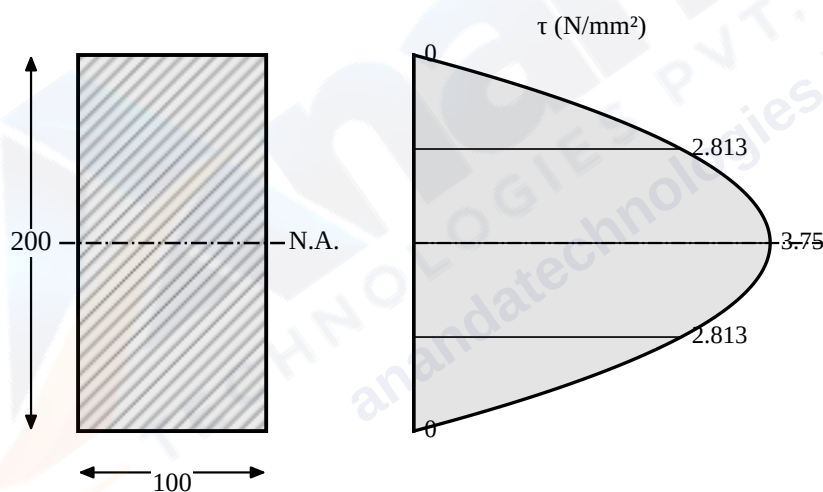
$$\tau_{avg} = F/A = 50 \times 10^3 / (100 \times 200) = 2.5 \text{ N/mm}^2$$

$$\tau_{max} = 1.5 \tau_{avg} = 1.5 \times 2.5 = 3.75 \text{ N/mm}^2 \text{ (at N.A.)}$$

At $y = 50$ mm above N.A.: $\tau = F \cdot A \cdot \bar{y} / (I \cdot b)$. $I = 100 \times 200^3 / 12 = 6.6667 \times 10^7 \text{ mm}^4$.

Area above the layer $A = 100 \times (100 - 50) = 5000 \text{ mm}^2$, $\bar{y} = 50 + 25 = 75 \text{ mm}$

$$\tau = 50 \times 10^3 \times 5000 \times 75 / (6.6667 \times 10^7 \times 100) = 2.8125 \text{ N/mm}^2$$



Shear stress distribution (N/mm²) — parabolic

(τ_{avg} 1 M, τ_{max} 1 M, τ at 50 mm 1 M, diagram 1 M)

c) Euler's and Rankine's formulae; effective lengths.

4 M

Euler's formula (crippling / buckling load of a long column):

$$P_E = \pi^2 EI_{min} / L_e^2$$

P_E = Euler's crippling load (N), E = modulus of elasticity (N/mm²), I_{min} = least moment of inertia of the section (mm⁴), L_e = effective length of column (mm).

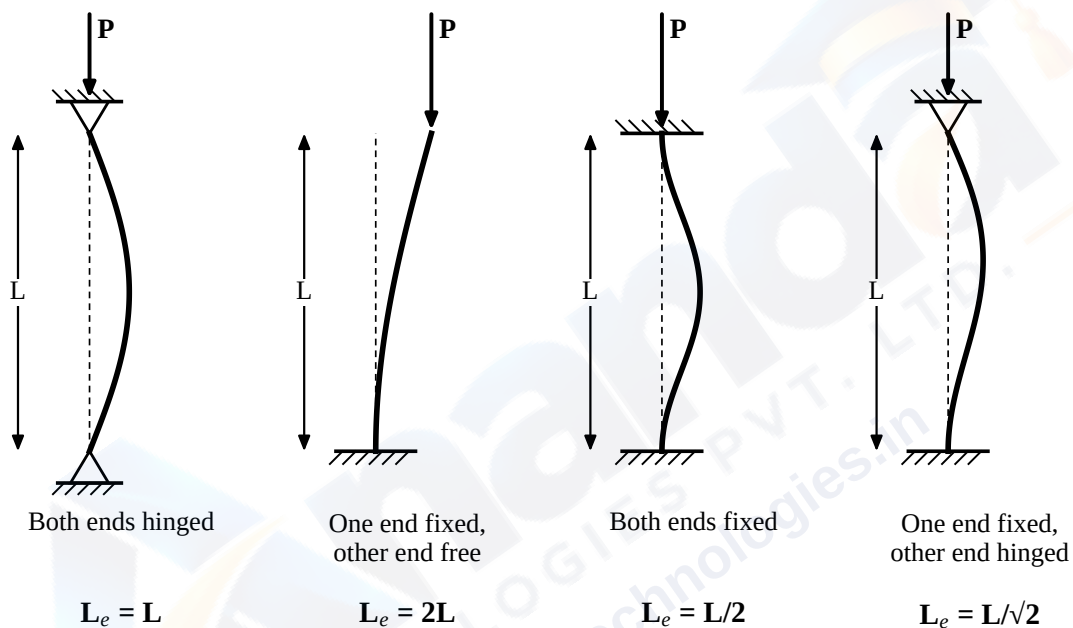
Rankine's formula (empirical, for short as well as long columns):

$$P_R = \sigma_c \cdot A / [1 + a (L_e/k)^2]$$

P_R = Rankine's crippling load, σ_c = crushing stress of the material, A = cross-sectional area, a = Rankine's constant ($= \sigma_c / \pi^2 E$, e.g. 1/7500 for mild steel, 1/1600 for cast iron), L_e = effective length, k = least radius of gyration $= \sqrt{I_{\min}/A}$, L_e/k = slenderness ratio.

End condition	Effective length L_e
Both ends hinged	L
One end fixed, other end free	$2L$
Both ends fixed	$L/2$
One end fixed, other end hinged	$L/\sqrt{2}$

(L = actual length of column)



Effective length of columns for different end conditions

(Euler 1 M, Rankine 1 M, effective lengths 2 M)

d) Total change in length of stepped bar.

4 M

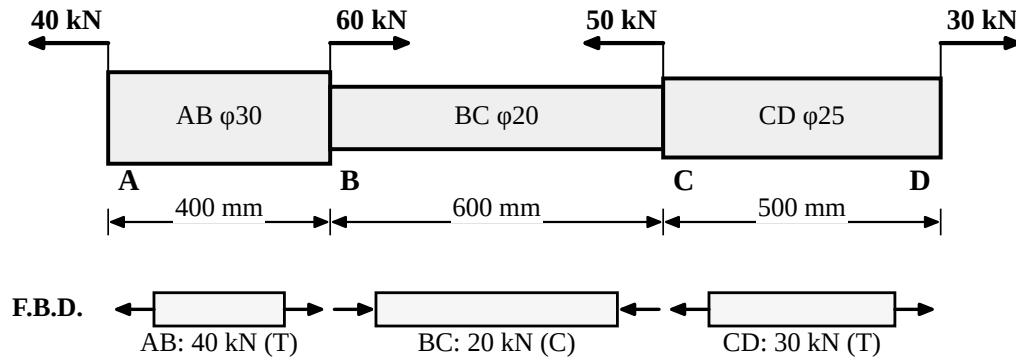
$E = 200 \text{ GPa} = 2 \times 10^5 \text{ N/mm}^2$. Check equilibrium: $-40 + 60 - 50 + 30 = 0 \checkmark$ (forces to the right +ve).

Internal forces (free body diagram of each part, considering forces to the left of a section; tension +ve):

Part	Axial force P	L (mm)	A (mm ²)	Stress $\sigma = P/A$ (N/mm ²)	$\delta L = PL/AE$ (mm)
AB	40 kN (T)	400	$\pi \times 30^2/4 = 706.86$	56.588	$+40000 \times 400 / (706.86 \times 2 \times 10^5) = 0.11318$
BC	20 kN (C)	600	$\pi \times 20^2/4 = 314.16$	-63.662	$-20000 \times 600 / (314.16 \times 2 \times 10^5) = -0.19099$

CD	30 kN (T)	500	$\pi \times 25^2 / 4 = 490.87$	61.115	$+30000 \times 500 / (490.87 \times 2 \times 10^5) = 0.15279$
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Total change in length $\delta L = \Sigma \delta L = 0.07498 \text{ mm}$ (increase)



Bar and free body diagrams of parts AB, BC, CD

(Internal forces 2 M, δL 2 M)

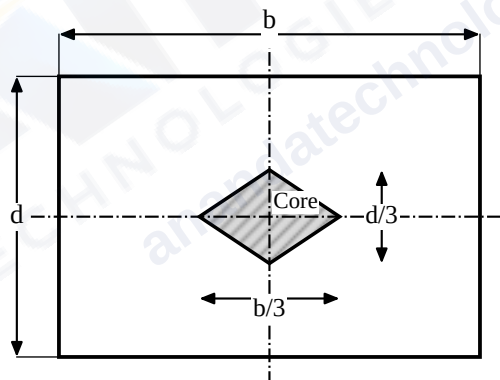
e) No-tension condition and core of rectangular section.

4 M

Condition of no tension: For an eccentrically loaded column, the resultant stress at the base will be entirely compressive (no tension) when the bending stress does not exceed the direct stress, i.e. $\sigma_b \leq \sigma_0 \Rightarrow P \cdot e / Z \leq P / A \Rightarrow e \leq Z / A$.

For a rectangular section $b \times d$ (load on the axis, eccentricity along b): $Z / A = (d b^2 / 6) / (b d) = b / 6$, so $e \leq b / 6$. Hence the load must lie within the **middle third** of the base (total width $b/3$).

Core (kernel) of section: The area around the centroid within which the load must act so that no tensile stress is produced anywhere in the section. For a rectangle it is a rhombus with diagonals $b/3$ and $d/3$.



Core (kernel) of a rectangular section — rhombus with diagonals $b/3$ and $d/3$

(Condition 2 M, sketch 2 M)

Q.5

a) Overhanging beam — SFD, BMD, contraflexure.

6 M

Support reactions (R_A and R_B act upward):

Taking moments about A, $\Sigma M_A = 0$:

$$R_B \times 5 = (10 \times 5) \times 2.5 + 15 \times 6.5 = 222.5$$

$$\therefore R_B = 222.5 / 5 = \mathbf{44.5 \text{ kN}}$$

$$\Sigma F_y = 0: R_A + R_B = 15 + 10 \times 5 = 65 \text{ kN}$$

$$\therefore R_A = 65 - 44.5 = \mathbf{20.5 \text{ kN}}$$

Shear force calculations (from the left, upward force on left = +ve):

$$\text{S.F. just right of A} = 20.5 = \mathbf{20.5 \text{ kN}}$$

$$\text{S.F. just left of B} = 20.5 - 10 \times 5 = \mathbf{-29.5 \text{ kN}}$$

$$\text{S.F. just right of B} = 20.5 + 44.5 - 10 \times 5 = \mathbf{15 \text{ kN}}$$

$$\text{S.F. just left of C} = 20.5 + 44.5 - 10 \times 5 = \mathbf{15 \text{ kN}}$$

Bending moment calculations (sagging = +ve, hogging = -ve):

$$M_A = 0 \text{ (simple support)}$$

$$M_B = 20.5 \times 5 - 10 \times 5 \times 2.5 = \mathbf{-22.5 \text{ kN}\cdot\text{m}}$$

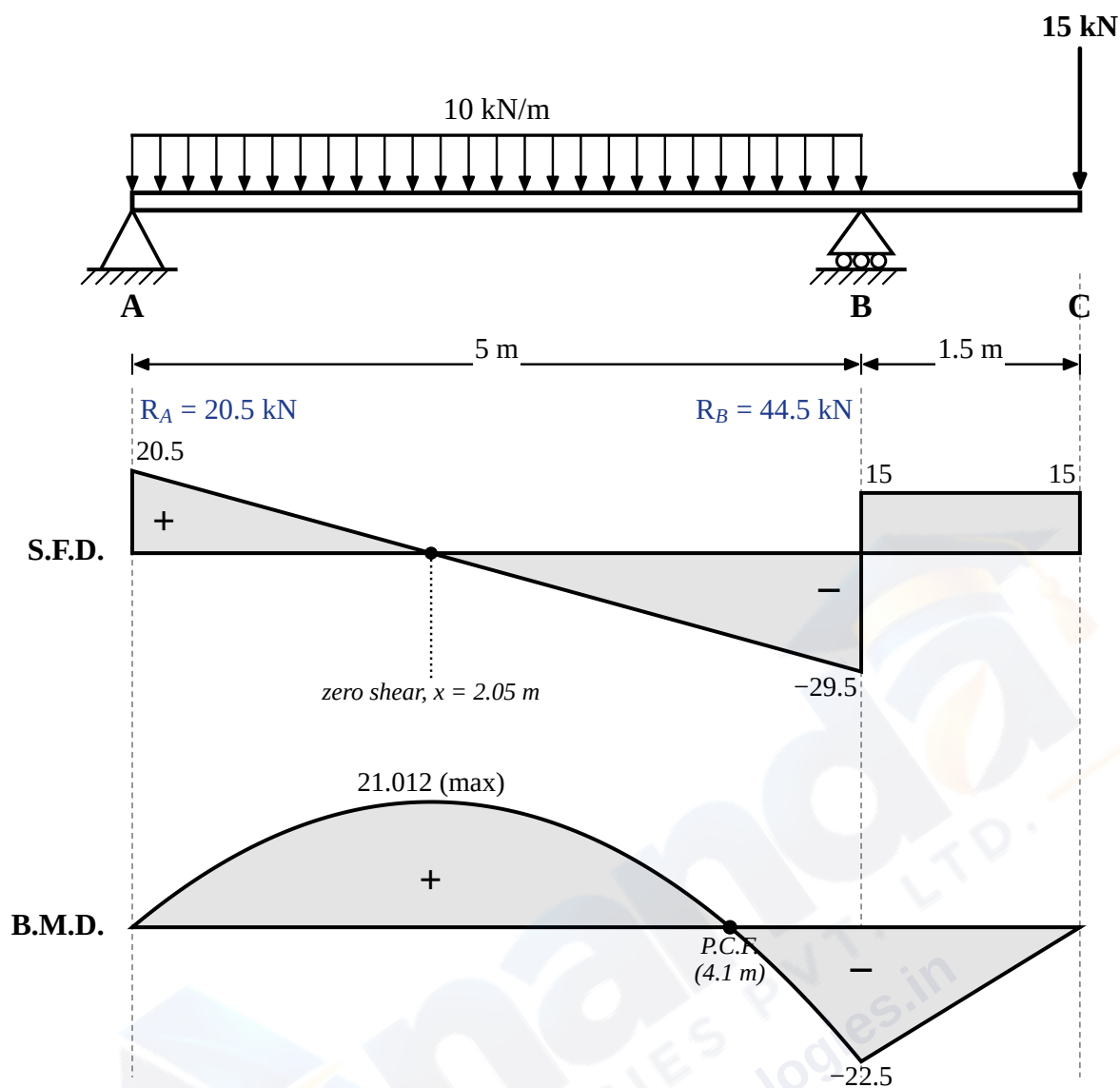
$$M_C = 0 \text{ (free end)}$$

Point of zero shear (contra-shear): let it be at x m from A. Between A and the next point S.F. = $20.5 - 10(x - 0) = 0 \Rightarrow x = \mathbf{2.05 \text{ m}}$ from A.

$$\text{Maximum B.M. at this section: } M_{\max} = 20.5 \times 2.05 - 10 \times 2.05 \times 1.025 = \mathbf{21.012 \text{ kN}\cdot\text{m}}$$

S.F. changes sign at B, so B.M. has a maximum (peak) value at B: $M_B = \mathbf{-22.5 \text{ kN}\cdot\text{m}}$

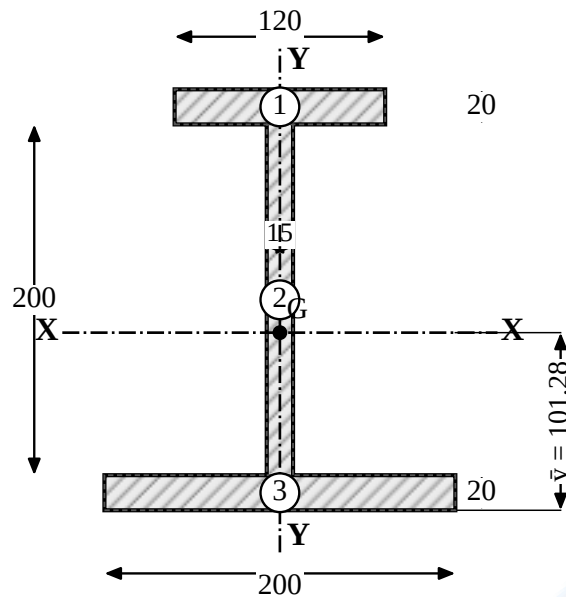
Point of contraflexure (B.M. changes sign between A and B): taking a section at x m from A, $M_x = -5x^2 + 20.5x = 0 \Rightarrow x = \mathbf{4.1 \text{ m}}$ from A.



S.F. and B.M. diagrams

b) MI of unsymmetrical I-section.

6 M



Unsymmetrical I-section (all dimensions in mm)

Centroid (reference: bottom edge; section symmetric about Y–Y so $\bar{x} = 100$ mm from left edge):

Part	a (mm ²)	y (mm)	a·y (mm ³)	I _{G own} (mm ⁴)	h = y – $\bar{y}$ (mm)	a·h ² (mm ⁴)
Top flange 120×20	2400	230	5.52×10^5	80000	128.72	3.9767×10^7
Web 15×200	3000	120	3.6×10^5	1×10^7	18.72	1.0517×10^6
Bottom flange 200×20	4000	10	40000	1.3333×10^5	–91.28	3.3326×10^7
Σ	9400		9.52×10^5	1.0213×10^7		7.4145×10^7

$$\bar{y} = \Sigma a \cdot y / \Sigma a = 9.52 \times 10^5 / 9400 = \mathbf{101.28 \text{ mm}}$$
 from bottom

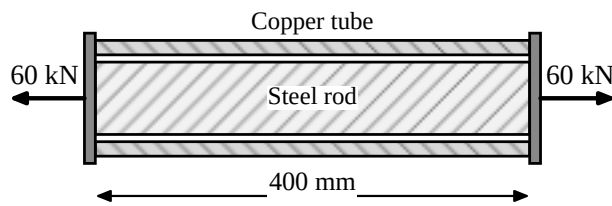
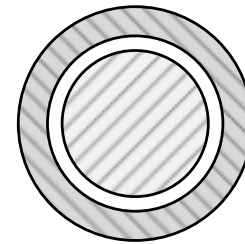
$$\mathbf{I_{XX} = \Sigma I_G + \Sigma a \cdot h^2 = 8.4358 \times 10^7 \text{ mm}^4}$$

$$\mathbf{I_{YY}}$$
 (all centroids lie on Y–Y, so h = 0): $I_{YY} = 20 \times 120^3/12 + 200 \times 15^3/12 + 20 \times 200^3/12 = \mathbf{1.627 \times 10^7 \text{ mm}^4}$

(Centroid 2 M, I_{xx} 3 M, I_{yy} 1 M)

c) Composite bar — steel rod in copper tube.

6 M



Section: rod $\phi 25$,
tube $\phi 40/\phi 30$

Composite bar under axial pull

$$A_s = \pi \times 25^2/4 = 490.87 \text{ mm}^2; A_c = \pi(40^2 - 30^2)/4 = 549.78 \text{ mm}^2$$

Condition 1 — equal strain (both fixed together, same length and same extension): $\sigma_s/E_s = \sigma_c/E_c$

$$\Rightarrow \sigma_s = (200/100) \sigma_c = 2\sigma_c$$

Condition 2 — load sharing: $P = \sigma_s A_s + \sigma_c A_c$

$$60 \times 10^3 = 2\sigma_c \times 490.87 + \sigma_c \times 549.78 = 1531.53 \sigma_c$$

$$\sigma_c = 39.177 \text{ N/mm}^2 \text{ (tensile); } \sigma_s = 2 \times 39.177 = 78.353 \text{ N/mm}^2 \text{ (tensile)}$$

Load on steel $P_s = 78.353 \times 490.87 = 38.462 \text{ kN}$; load on copper $P_c = 39.177 \times 549.78 = 21.538 \text{ kN}$ (sum = 60 kN ✓)

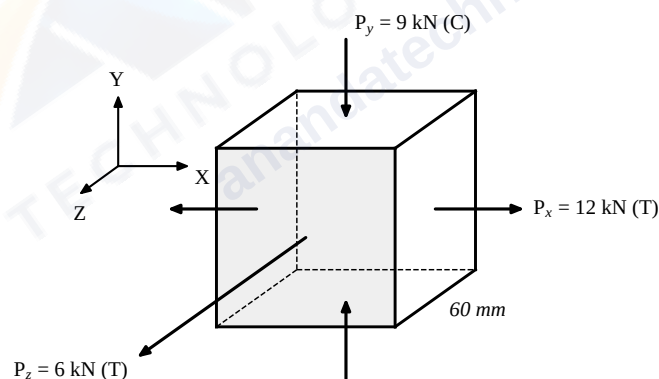
$$\text{Extension } \delta L = \sigma_s L/E_s = 78.353 \times 400 / (2 \times 10^5) = 0.15671 \text{ mm}$$

(Equations 2 M, stresses 2 M, loads 1 M, extension 1 M)

Q.6

a) Cube under tri-axial forces — change in volume.

6 M



Cube of 60 mm side under tri-axial loading

$$\text{Area of each face} = 60 \times 60 = 3600 \text{ mm}^2$$

$$\sigma_x = 12000/3600 = 3.3333 \text{ N/mm}^2 \text{ (T)}; \sigma_y = 9000/3600 = 2.5 \text{ N/mm}^2 \text{ (C)}; \sigma_z = 6000/3600 = 1.6667 \text{ N/mm}^2 \text{ (T)}$$

Strain in each direction (tension +ve): $e_x = [\sigma_x - \mu(\sigma_y + \sigma_z)]/E$, etc.

$$e_x = [3.3333 - 0.3(-2.5 + 1.6667)] / (2 \times 10^5) = \mathbf{17.9167 \times 10^{-6}}$$

$$e_y = [-2.5 - 0.3(3.3333 + 1.6667)] / (2 \times 10^5) = \mathbf{-20 \times 10^{-6}}$$

$$e_z = [1.6667 - 0.3(3.3333 + -2.5)] / (2 \times 10^5) = \mathbf{7.0833 \times 10^{-6}}$$

$$\text{Volumetric strain } e_v = e_x + e_y + e_z = \mathbf{5 \times 10^{-6}}$$

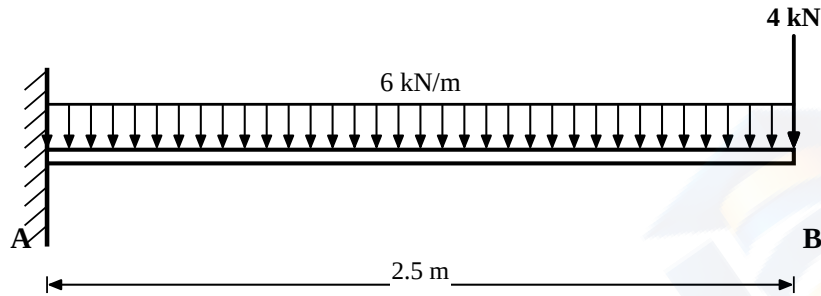
$$(\text{Check: } e_v = (\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)/E = 2.5 \times 0.4 / (2 \times 10^5) = 5 \times 10^{-6} \checkmark)$$

$$\text{Change in volume } \delta V = e_v \times V = 5 \times 10^{-6} \times 60^3 = \mathbf{1.08 \text{ mm}^3} \text{ (increase)}$$

(Stresses 1 M, strains 3 M, e_v 1 M, δV 1 M)

b) Design of rectangular cantilever section.

6 M



Cantilever with loads

$$\text{Maximum B.M. occurs at the fixed end: } M = wL^2/2 + W \cdot L = 6 \times 2.5^2/2 + 4 \times 2.5 = 18.75 + 10 = \mathbf{28.75 \text{ kN}\cdot\text{m}} = 2.875 \times 10^7 \text{ N}\cdot\text{mm}$$

$$\text{Let width} = b, \text{ depth } d = 2b. \text{ Section modulus } Z = bd^2/6 = b(2b)^2/6 = 4b^3/6 = (2/3) b^3$$

$$\text{Flexure formula: } M = \sigma \cdot Z \Rightarrow 2.875 \times 10^7 = 10 \times (2/3) b^3 \Rightarrow b^3 = 4.3125 \times 10^6 \text{ mm}^3$$

$$\mathbf{b = 162.77 \text{ mm}} \text{ and } \mathbf{d = 2b = 325.54 \text{ mm}}. \text{ Adopt a section of } \mathbf{165 \text{ mm} \times 330 \text{ mm}}.$$

$$\text{Check: } Z = 165 \times 330^2/6 = 2.9947 \times 10^6 \text{ mm}^3; \sigma = 2.875 \times 10^7 / 2.9947 \times 10^6 = 9.6 \text{ N/mm}^2 < 10 \checkmark$$

(M_{max} 2 M, Z 1 M, b and d 3 M)

c) Hollow circular column — eccentric load.

6 M

$$\text{Given: } D = 250 \text{ mm}, d = 200 \text{ mm}, P = 150 \text{ kN}, e = 60 \text{ mm}.$$

$$A = \pi(250^2 - 200^2)/4 = 17671.46 \text{ mm}^2$$

$$I = \pi(250^4 - 200^4)/64 = 1.1321 \times 10^8 \text{ mm}^4$$

$$Z = I/(D/2) = 1.1321 \times 10^8 / 125 = 9.0566 \times 10^5 \text{ mm}^3$$

$$\text{Direct stress } \sigma_0 = P/A = 1.5 \times 10^5 / 17671.46 = \mathbf{8.488 \text{ N/mm}^2} \text{ (C)}$$

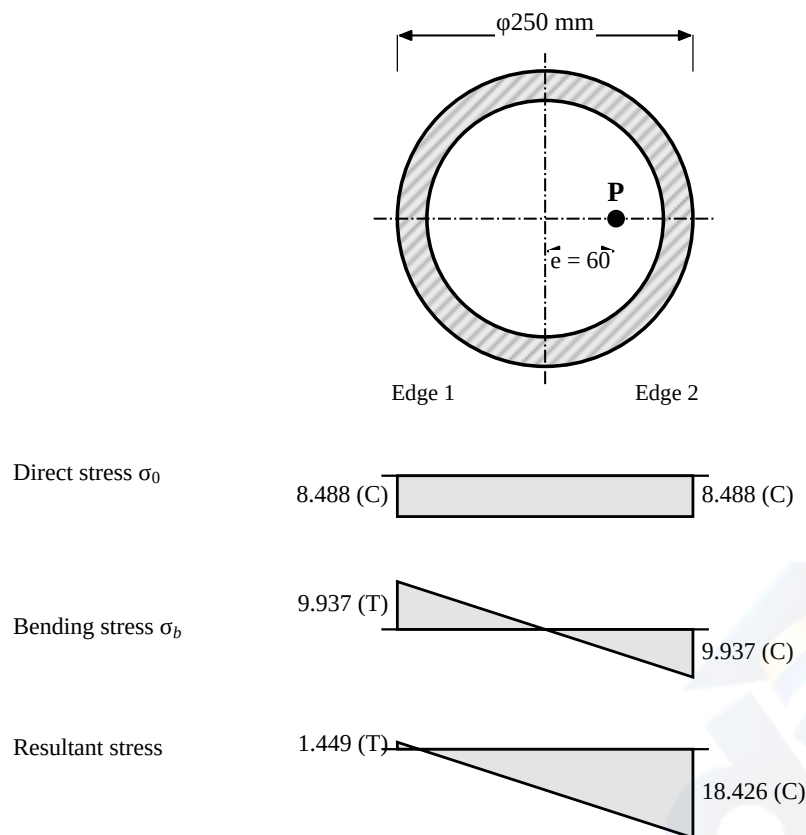
$$\text{Bending moment } M = P \cdot e = 1.5 \times 10^5 \times 60 = 9 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\text{Bending stress } \sigma_b = M/Z = 9 \times 10^6 / 9.0566 \times 10^5 = \mathbf{9.937 \text{ N/mm}^2}$$

$$\sigma_{max} = \sigma_0 + \sigma_b = 8.488 + 9.937 = \mathbf{18.426 \text{ N/mm}^2} \text{ (compressive)}$$

$$\sigma_{min} = \sigma_0 - \sigma_b = 8.488 - 9.937 = \mathbf{-1.449 \text{ N/mm}^2} \text{ i.e. } \mathbf{1.449 \text{ N/mm}^2} \text{ tensile}$$

Limit of eccentricity for no tension: $e_{max} = Z/A = 9.0566 \times 10^5 / 17671.46 = \mathbf{51.25 \text{ mm}}$. Since $e = 60 \text{ mm} > 51.25 \text{ mm}$, tension develops at the edge away from the load.



(stresses in N/mm^2 ; C = compression, drawn below the line; T = tension, drawn above)

Fig.: Section with load position and stress distribution

Core of hollow circular section: circle of radius $(D^2 + d^2)/(8D) = 51.25$ mm about the centre.

(A and Z 1 M, σ_0 1 M, σ_b 1 M, $\sigma_{max}/\sigma_{min}$ 1 M, diagram 1 M, limit 1 M)

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